The extensive and intensive margin of price adjustment to cost shocks: Evidence from Danish multiproduct firms *

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Abstract

This paper studies price adjustment in a novel monthly dataset of individual product prices of multiproduct firms, merged with firm-level balance sheets and cost data. According to the theoretical literature on price setting, the interdependence between state-dependence in the decision to whether or not change prices (extensive margin) and the actual amount by which prices change (the intensive margin) is key in determining the real effects of monetary policy. We estimate price adjustment to shocks to firm-level import and energy costs (due to oil supply shocks) along extensive and intensive margins, accounting for endogenous selection bias due to state-dependent pricing with a two-step econometric approach. In the first step, we estimate the probability of price increases and decreases (over horizons up to 2 years) using a multinomial logit model. There is evidence of synchronization of price change decisions within firms, especially as their number of goods increases, in line with price-setting models of multiproduct firms. We also find evidence of state dependence as cost shocks affect the likelihood of price changes. Nevertheless, state-dependence translates only into a statistically significant but economically small selection bias in pass-through. The bulk of overall price adjustment to our shocks is consistent with hybrid state-dependent (menu cost) models with a significant degree of time dependence. Finally, we find that pass-through of energy and import cost shocks is quite heterogeneous across sectors and firms. Gradual adjustment to energy costs mainly reflects delayed price responses in final goods and less oil intensive sectors, in line with pipeline pressures along the supply chain. Especially for import cost shocks, pass-through of larger firms is lower than that of smaller firms, suggesting a role for strategic complementarities in the intensive margin of price adjustment.

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1 Introduction

Price adjustment by firms is lumpy: individual good prices alternate between long spells in which they are unchanged, and large (but also small) increases and decreases, mostly idiosyncratic, in "reset" prices. State-of-the-art macro models of firms' price setting stress the relevance of lumpiness and heterogeneity in shaping aggregate inflation determination. Specifically, the theoretical literature on price setting has pointed out that the interdependence between the decision to whether or not change prices (often dubbed the extensive margin) and the actual amount by which prices change (often dubbed the intensive margin) contributes to shape the real effects of monetary policy. Menu costs models of multiproduct firms have been shown to be able to generate empirically plausible real effects of monetary policy because of within-firm price synchronization (Alvarez and Lippi, 2014) or many small idiosyncratic cost shocks (Midrigan, 2011); this is especially so when these models also feature some degree of time-dependence in the decision to change prices (Alvarez et al., 2016). All these mechanisms attenuate the "selection bias" due to the interaction between state-dependence in the decision to change prices and the intensive margin of price adjustment. Namely, selection bias arises since the prices which are more likely to change endogenously are those farther from their desired level, so that reset prices display on average large(r) changes condi-tional on a cost shock (than what would be implied by time dependence).[1] Microeconomic evidence on actual price decisions of (multiproduct) firms is thus crucial to better understand the monetary transmission and aggregate inflation determination.

This paper contributes to the literature on price setting by estimating price adjustment along the extensive and intensive margin in a novel monthly dataset of prices of multiproduct firms, merged with firm-level balance sheet and cost data (including monthly labor costs and intermediates). To the best of our knowledge, this is the first paper that accounts for endogenous selection bias due to state-dependent pricing decisions in estimating pass-through into prices of firm-level cost shocks. Specifically, it relies on two-step econometric techniques (pioneered in labor economics by Heckman (1979)) developed in Bourguignon et al. (2007) — and that we adapt to a dynamic setting to estimate the adjustment of good-level prices over time to energy (due to oil supply shocks) and import costs. We use producer price micro data from the monthly survey underlying the producer price index (PPI) built by the Danish statistical office.[2] A crucial feature of the data that makes it relevant to an analysis of pricing by multiproduct firms is that there is substantial variation in the

[1]Specifically, in the general class of state-dependent pricing models with random menu costs studied by Alvarez and Lippi (2021), we show below that selection bias conditional on changing prices in response to a permanent cost shock is increasing in the degree of state-dependence in the decision to change prices.

[2]See Nakamura and Steinsson (2008) for a description of US PPI data; PPI microdata of other European countries were analyzed in Vermeulen et al. (2012).

number of goods across more than 1,000 firms. This allows us to study how price-setting features vary with the number of goods. Moreover, PPI micro data are especially useful to analyze in light of the above theoretical literature, as noted already by Bhattarai and Schoenle (2014), since they are consistent with the basic assumptions of virtually all price-setting models in macroeconomics, where it is producing firms that set prices (rather than retailers whose prices are comprised in the CPI).[3] In contrast to most CPI data, a further advantage of our dataset is that we can link prices to balance sheet and cost data at the firm level.[4]

Our main results are as follows. In the first step, we estimate the likelihood of price changes (over horizons up to 2 years) by using a flexible multinomial logit model. We find evidence in support of (partial) synchronization of adjustment decisions within firms, especially as the number of goods they produce increases. Namely, within a multiproduct firm the probability that a given price increases (decreases) is higher the larger the fraction of other prices that are decreasing (increasing).[5] We also find evidence of state dependence, as the likelihood of price adjustment over time is affected not only by our cost shocks, but also by aggregate time series such as CPI inflation and (even) the exchange rate.

In line with this evidence of state dependence, we find that in our second step we cannot reject the hypothesis of no selection bias, thus validating our econometric approach. Nevertheless, building on the decomposition of (conditional) inflation dynamics by Caballero and Engel (2007), we document that price adjustment consistent with time-dependence (equal to our estimates of pass-through corrected for selection bias times the unconditional probability of price changes at each horizon) accounts for the bulk of the overall price response to our shocks (estimated over zero and non-zero price changes). Overall, our results on the extensive and intensive margin of price adjustment support hybrid state-dependent models of multiproduct firms with significant attenuation in selection due to a combination of time dependence and some degree of imperfect synchronization in price changes (e.g. where firms' fixed costs of price reoptimization are partly common across goods). Remarkably, this (conditional) evidence fits in well with the fact that in our micro price data the unconditional size distribution of price changes is quite leptokurtic, consistent with theoretical distributions arising in the above hybrid models of multiproduct firms (see Alvarez

[3]A similar analysis of producer pricing decisions is not feasible with CPI data since the CPI sampling procedure maps to stores, so-called "outlets", which may sell goods from any number of firms, including imports. This makes pricing a complicated web of decisions that involves the whole distribution network.

[4]A second advantage of PPI micro data, relative to consumer prices, is that they contain very few "sales" prices (namely very short-lived price changes that are quickly reverted, see e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2008)). For this reason, PPI microdata do not necessarily call for ad-hoc "filtering" to make them amenable to interpretation through the lens of standard price setting models. This is especially useful in econometric analyses like ours.

[5]In Section 2 we document that key descriptive properties of price dynamics across firms, such as frequency and size of price changes, are broadly invariant to the number of goods firms produce, in contrast with the full synchronization predicted by multiproduct firm models with fixed costs of changing prices common across goods.

et al. (2016)).[6]

Finally, we document that pass-through of reset prices to shocks to import and energy costs is quite heterogeneous across sectors and firms of different size (as proxied by the number of employees). Specifically, reset prices adjust to import cost shocks immediately, while their reaction to energy cost shocks is gradual over time (see also Ganapati et al. (2020)). This gradual adjustment in reset prices mainly reflects heterogeneity in the firms' sector position in the supply chain and in the sectoral intensity of (both direct and indirect) use of oil: reset prices of goods in final sectors and especially sectors with a low intensity in oil use adjust with a substantial delay. These results provide novel micro-based evidence on the debate about the inflationary dynamics of idiosyncratic and (relatively more) common shocks (see e.g. Boivin et al. (2009)). Firm-specific import cost shocks elicit a faster adjustment than energy cost shocks, whose effects instead gradually build up through different sectors along the supply chain, in line with the pipeline pressures view (see e.g. Smets et al. (2018) or Duprez and Magerman (2019)). We also find evidence that in response to import cost shocks, smaller firms (with fewer employees) adjust reset prices in the medium run by more than larger firms, which is in line with the literature stressing the role of strategic complementarities for the latter (e.g. Amiti et al. (2019)).

The rest of the paper is organized as follows: Section 2 describes our datasets (while details are relegated to the appendix) and presents key descriptive statistics on extensive- and intensive-margin price changes of multiproduct firms. Section 3 reviews price adjustment under sticky prices, and lays out the methodology we use to estimate structural pass-through coefficients in a way that accounts for both sticky prices and strategic complementarities. Section 4 discusses the results of our empirical analysis of two persistent cost shocks: a oil supply-driven shock to firm-level energy costs, as well as idiosyncratic import cost shocks at the firm level.

2 Data and some descriptive statistics

Before turning to our investigation of price adjustment in response to structural cost shocks, we find it useful to provide a description of our dataset. The main part of the data we compile consists of the (confidential) microdata underlying the Danish producer price index from 1993 to 2017. In our analysis, we will leverage the fact that we can link the producer price data to high-frequency statements on sales and costs, including labor costs. However, the availability of high-frequency payroll data only between January 2008 and June of 2017 effectively limits the sample of our estimations of pass-through. In this section we report useful descriptive statistics on unconditional

[6] Interestingly, we also find that firm-level variable costs are similarly leptokurtic, with a large proportion of very small cost changes, in line with assumptions in the models of Midrigan (2011) and Karadi and Reiff (2019).

price adjustment in our dataset of multiproduct firms, following Bhattarai and Schoenle (2014). In contrast to the latter paper, we find that across Danish firms with different numbers of goods there are very few differences in aggregate statistics on price adjustment, such as frequency, size, direction, and dispersion of price changes. These findings are consistent with some specifications of fixed costs of changing prices at the firm level in Alvarez and Lippi (2014) and Bonomo et al. (2019), where those costs increase with the number of prices which are reoptimized, rather than being constant across them.

2.1 Producer prices

The Danish PPI contains monthly price quotes of actual transactions for 558 different product categories, that is, particular sets of individual items defined in the 8-digit codes according to the Harmonized Commodity Description and Coding Systems (HS). At the firm-good level, we track 5,354 goods for both domestic sales and exports. The most important firms within selected areas are requested to report prices in order to ensure that the producer price index covers at least 70% of Danish total production. Appendix A describes the multi-stage sampling design.

Renkin and Hviid (2020) have used the data to study the pass-through of labor cost to prices, but ours is the first paper that uses this dataset to analyze the interaction between price stickiness and pass-through. Therefore, and to benchmark moments of the data against the U.S. PPI more commonly used in the literature, we first document key characteristics of the panel.[7] Note that we do not observe quantities, so we use equal weights of goods within firms and categories wherever needed.

2.1.1 Multiproduct firms

The PPI data allow us to identify firms according to the number of goods they produce. Using the firm identifier, we are able to determine the number of goods reported by a firm in a given month, and to the extent that this is representative for the total number of goods produced, put special emphasis on multiproduct firms in the analysis. Following Bhattarai and Schoenle (2014), we then allocate the firms to five bins according to the mean of products reported over the sample period. The cutoffs used on the mean number of products are 1, 3, 5, and 7. Namely, in the first bin we include all firms that report on average only the price of one product, in the second bin those the

[7] Two key differences relative to the US PPI data used in the literature are that first, Danish PPI prices are collected at the firm/enterprise level rather than the establishment level ("price-forming units" usually defined to be "production entities in a single location", by the BLS); and second, that both domestic and export prices are reported. Both features of the data imply that relying on the US PPI micro data may actually lead to underestimating the number of products at the firm level.

report on average up to the prices of three products and so on until the fifth bin, in which we include firms with more than 7 products.

Table 1 presents descriptive statistics on the distribution of firms and products across these five bins. The distribution of products across firms is comparable to that in the US PPI dataset, with the exception that the share (and standard deviation) of the number of goods of firms in bin 5 (with the highest number of products) are substantially larger; i.e. we observe many multiproduct firms. Observe that the Danish data contains about 1,000, compared to more than 28,000 in the US PPI.

The table also shows that while the majority of firms, around 80%, falls in the first three bins, i.e. on average reports up to five product prices, firms in bins 4 and 5 produce many more goods, so that they account for a much larger share of prices than of firms. Specifically, firms in bins 4 and 5 set more than 50% of all prices in our data, again comparable with U.S. PPI data. The distribution across bins is robust to only including goods sold in the domestic market. When grouping the firms according to the number of domestic goods they sell, goods of firms with up to 3 products represent a larger share of our sample, but prices set by firms with 5 or more products still make up 40% of the dataset.

Finally, Table 1 also reports statistics regarding firm size (mean and median employment in full-time equivalents during the accounting year) at the firm-level and divided by the number of goods. Clearly, in line with the results in Bhattarai and Schoenle (2014), firms producing more goods do not have more employees per good (but for those in the last bin); however they are overall larger than firms producing less goods.

2.1.2 Frequency of price adjustment

Our price observations are actual transaction prices, as requested by Danmark Statistics. We first compute frequencies as the mean fraction of price changes during the life of a good. For exported goods, we define as a price change if both the value in Danish kroner and in the currency in which the price is reported change, if the two differ. Also, we do not explicitly take into account issues of left-censoring of price-spells. For our purpose, it is most relevant that we apply our method consistently across all firms. The mean adjustment frequency across all goods for the subsamples are depicted in the third panel of Table 1. The median (mean) adjustment frequency in the sample is 8.00% (20.6%), corresponding to a median implied duration of a price spell of 12 months. Overall price adjustments are therefore slightly less frequent than in the U.S. PPI (whose median frequency is 10.8% in Nakamura and Steinsson (2008)) but very close to comparable euro area statistics Vermeulen et al. (2012).

Table 1: Summary statistics by number of products

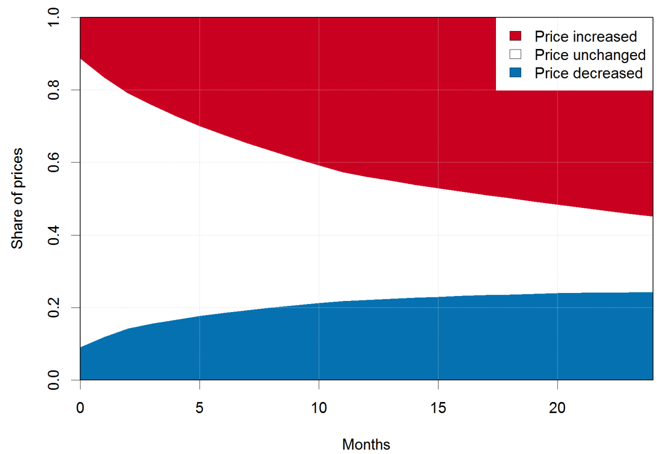
	All	1	1-3	3-5	5-7	7+
No. of firms	942	92	449	200	118	83
Mean employment (FTE)	630.5	76.4	168.4	259.2	249.3	1601.8
Median employment (FTE)	161.5	44.1	62.4	134.9	146.8	534.9
Mean employment per good	70.5	76.4	66.6	63.5	44.2	96.6
Median employment per good	32.9	44.1	25.1	34.1	24.9	51
Mean age (years)	33.5	31.5	29.6	34.1	32.0	37.5
Median age (years)	29.0	28.0	28.0	31.0	26.0	32.0
Share of total prices	100.0	1.3	20.5	22.2	18.5	37.5
Mean no. of products	9.0	1.0	2.7	4.1	5.8	19.4
Std. err. no. of products	12.9	0.0	0.5	0.6	0.5	18.6
25th percentile	3.0	1.0	2.5	3.6	5.4	8.8
Median	5.1	1.0	3.0	4.1	5.8	11.6
75th percentile	8.7	1.0	3.0	4.6	6.0	16.9
Mean adj. frequency across goods	20.6	22.6	18.4	20.3	16.4	24.2
Median adj. frequency across goods	8.0	8.1	6.1	8.0	7.1	10.0
Mean adj. freq., median good	17.9	22.1	17.6	18.5	14.2	18.9
Median adj. frequency, median good	7.0	8.0	6.3	7.7	6.8	8.8
Mean fraction of increases	68.0	67.7	67.6	67.5	70.8	67.6
Mean abs. size of price adj.	6.2	5.8	6.6	5.5	6.1	7.1
Increases only	6.0	5.7	6.3	5.4	5.7	6.6
Decreases only	7.4	6.0	7.2	7.8	7.3	8.2
Kurtosis	4.9	4.5	5.0	4.9	5.0	4.8

Note: Summary statistics on distribution of firms and prices across distinct bins of the average number of product reported between January 2008 and December 2017. Frequencies are reported in % per month, and computed as in Bhattarai and Schoenle (2014): Take the mean of adjustment frequencies at the good level, then compute the median frequency of price changes across goods in a firm. Finally, we report the mean and median across firms in a given subsample. Fractions are reported in percentages. We report price change statistics by broad economic categories in the data appendix.

Table 1 further documents that neither the frequency nor the size of price changes depend on the number of products produced. We proceed as in Bhattarai and Schoenle (2014) and aggregate goods within multiproduct firms by taking the median of good-level price change frequencies, and then report key moments of the firm-level distribution in the third panel of Table 1.[8] While overall frequencies (reported in the column dubbed "All") are comparable to those obtained from the U.S. PPI, as we have already noted, the table displays no monotone or statistically significant relationship between the number of goods produced and price adjustment statistics. Differently from the US case, across all bins, the median adjustment frequency is between 6.1% for bin 2 and 10% for bin 5, but the second lowest value is registered for bin 4. Moreover, over two-thirds of these changes

[8] In the data appendix, we include replications of figures presented in Bhattarai and Schoenle (2014) in which we include 95% confidence intervals on all these statistics.

Figure 1: Frequency of cumulated price changes



Note: For every horizon h, this figure depicts the probability of having changed (increased or decreased) the price between month 0 and h.

(over all non-zero price changes) are price increases across all bins. Firms thus also adjust prices upward with similar frequency independently of the number of goods they produce.[9]

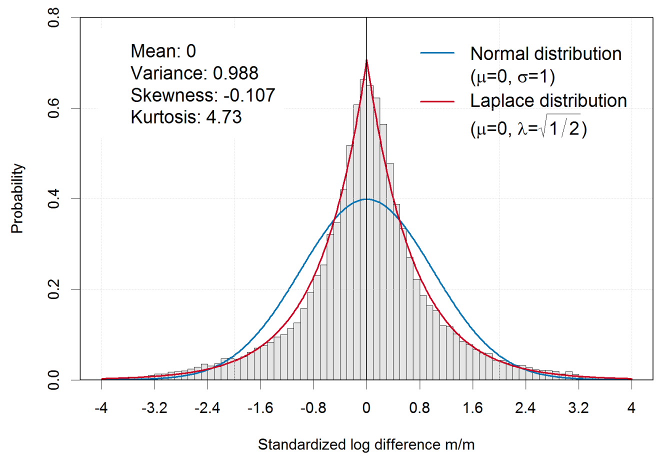
Since we are interested in dynamic pass-through, we report the unconditional frequencies for cumulative price changes in Figure 1; as we detail in Section 3, we will use these unconditional frequencies to estimate the contribution of the intensive margin to price adjustment condtional on our costs shocks. We cumulate (log) price changes over a period of up to 2 years and report in the figure, for every month, the share of prices that have increased or decreased. Figure 1 re-emphasizes the notion of price stickiness in the data: More than 30% of price spells remain unchanged after 12 months, and 20% even survive at least 24 months. Interestingly, the survival rate over time is higher than what would be implied by a simple exponential model based on a 20% monthly frequency of price changes.

2.1.3 Size and distribution of price changes

Next we report the size of price changes, defined as the absolute log difference of monthly price observations, conditional on a price change. Again, we compute this at the good level, take the median across goods in a firm, and then report the mean across firms. Table 1 (bottom panel) shows that the typical price change observed is around 6.2%. Decreases tend to be marginally larger than increases, 7.4% against 6%. We do not find, however, that the size of price increases

[9]Two notable differences with US PPI data could explain these discrepancies. First, the Danish PPI includes export goods, but conditioning on domestically sold goods only does not change this results qualitatively. Second, the U.S. PPI data is reported at the establishment level, whereas our data is reported by the firm.

Figure 2: Histograms of standardized price changes



Note: Price changes are the log difference in price, standardized by good category (first two digits of the product HS code). Price changes equal to zero or smaller than 0.1% are discarded. A normal and Laplace distribution with unit variance are superimposed.

or decreases vary by the number of products sold by the firm; if anything firms in bin 5 tend to display the largest price changes.[10]

Finally, we report in Figure 2 the distribution of non-zero price changes across all firms. This distribution is a typical target of quantitative price setting models with idiosyncratic firm shocks, since (Golosov and Lucas, 2007). To account for heterogeneity across goods, we follow (Alvarez et al., 2016) and standardize price changes by the 2-digit HS code level, and even exclude price changes smaller than 0.1%, to account for possible measurement error. The distribution of non-zero price changes has more mass around zero than would be implied by a normal distribution. It's kurtosis is 4.9 and thus closer to a Laplace distribution (with a kurtosis of 6). Overall, we also find that the distribution of price changes is broadly similar independently of the number of goods produced by a firm.[11] Consistenly, the last row of Table 1 shows that kurtosis is 4.5 for firms reporting only one price and between 4.8 and 5 across all other bins. The large mass of small price changes observed in our data is not consistent with purely state-dependent theories of price setting. Interestingly, the excess kurtosis present in the Danish PPI and the overall distribution of price changes is consistent with models with random menu costs and multiproduct firms setting several (4 or more) product prices, studied in Alvarez et al. (2016).

[10]In the data appendix, we include replications of figures presented in Bhattarai and Schoenle (2014) in which we include 95% confidence intervals on the statistics of adjustment size for each bin.

[11]All summary statistics such as the mean and median frequency and size of adjustment by product categories are reported in Appendix D.1

2.2 Firm-level cost data

We merge the PPI survey with firm-level data on the cost structure of production using a masked firm identifier. First, data from VAT filings contains information on nominal values of total sales and exports, as well as the purchases of foreign and total intermediate inputs. Second, we merge data from annual accounting statistics in Danish private-sector firms and information on firm age and size from business registers. The accounting statistics give us a complete picture of all the firm's cost structure at the annual frequency. Ultimately, we have access to monthly payrolls the firm pays to all its employees. The availability of the payroll data dictates the time span (2008-2017) used in the following econometric analysis.

We measure variable costs as the sum of domestic and imported intermediate goods purchased according to the VAT reporting, and the monthly wage bill. Comparing the distribution of firms prices and variable costs is useful, as several theories show that the latter are crucial to account for the aggregate effects of nominal shocks. Figure 17 in appendix B shows the standardized distribution of changes in variable costs with a superimposed normal and Laplace distribution with unit variance. The following findings stand out. First, contrary to prices, there are very few zero cost changes in our sample. Second, the distribution of cost changes has an even higher kurtosis than the price change distribution, with a larger incidence of small changes. Finally, we also show in the appendix the distribution of variable cost changes is less dispersed and has higher kurtosis in firms selling more products.

In this paper we focus on two different kinds of cost shocks, the first one with a predominantly idiosyncratic component, i.e. a shock to firm-specific prices of imported inputs; the second one with a significant common component across firms, namely oil supply shocks which we rescale proportionally to their effect on the price of energy in Denmark. To obtain a measure in terms of the firm's marginal costs, we interact the change in the respective input cost with the lagged intensity of the firm's cost structure in the respective input.

First, the energy cost shock is constructed as follows: We compute, on the one hand, the share of energy to total cost, which is reported in the annual accounting statistics. The energy share includes, apart from the expenditure on refined oil and petroleum, also electricity and heating. We provide histograms of the firm distribution of cost shares in the data appendix A, showing that the average share of cost spent on energy is relatively small, around 2%. To obtain a measure of energy marginal costs, we would need then to multiply the firm-level share with its price, but we don't observe it at the firm-level. We only observe an aggregate price of energy in Denmark, which however is likely to reflect many endogenous factors, including demand-side drivers. Therefore, to address these endogeneity concerns, for all firms we use instead the series of exogenous innovations in global oil supply by Baumeister and Hamilton (2019). Therefore, these shocks provide a source of fluctuations in energy costs that are exogenous to the Danish economy but also common across firms, to which firms have different direct exposure given by their energy intensity. Appendix B shows in details that these oil supply shocks have significant predictive power for the price of energy in Denmark and explain a substantial part of its variation. If an exogenous oil supply shock increases oil prices by 15%, the index of energy prices in Denmark moves up by 5%, leading to a cost push shock of 0.1% for a firm that has an average energy intensity in production.

Second, import shares are computed using the VAT reports, by dividing the total value of imports in a given month by total cost. The changes in import prices are directly observed in the import wave of the PPI data. Since we do not observe product-level weights, we take a geometric (mean) average of (log) import cost changes in DKK of all goods imported by the firm in a given month. If the firm does not purchase abroad, we set this to zero. As we explain in more detail below, we enhance identification of firm-specific import shocks by controlling for many other observable firm level costs and also by the prices of each firm's competitors.

3 Estimation of dynamic price adjustment under sticky prices

In this section we briefly review some useful theoretical results on lumpy price adjustment, starting with the case when firm prices are fully flexible, and then looking at the case of time- and state-dependent price stickiness. We use these results to guide our empirical analysis below.

3.1 Cost pass-through under price flexibility: Intensive margin

Let p_{ijt} be the log price of one (of possible many) good i in firm j. The general price setting equation under imperfect competition for the (static) optimal (log) price p^*_{ijt} postulates that it is a function of a possibly good-specific markup (μ_{ijt}) over marginal costs (mc_{ijt}):

$$p^*_{ijt} = mc_{ijt} + \mu_{ijt}. \tag{1}$$

We can define structural cost pass-through by taking the following total differential:

$$dp^*_{i,j,t} = dmc_{i,j,t} + \frac{\partial \mu_{i,j,t}}{\partial p^*_{i,j,t}} dp^*_{i,j,t}; \tag{2}$$

clearly, if the markup falls with p^*_{ijt}, i.e. $\frac{\partial \mu_{i,j,t}}{\partial p^*_{i,j,t}} < 0$, structural pass-through to a change in marginal costs will be incomplete and less than 1. The markup can depend on p^*_{ijt} for several reasons, for instance because the demand price elasticity is not constant, or because of imperfect competition (see the survey by Arkolakis and Morlacco (2017)).

Under fairly general conditions, including separability of the firm-level demand for each product, (Amiti et al., 2019, henceforth AIK) show that markups are a function of marginal costs and competitors' prices $\overline{p}_{i,-j,t}$, so that in first differences we obtain the following pricing relation:[12]

$$\Delta p^*_{i,j,t} = \frac{1}{1+\Gamma}\Delta mc_{i,j,t} + \frac{\Gamma}{1+\Gamma}\Delta\overline{p}_{i,-j,t}, \quad \Gamma = -\frac{\partial \mu_{i,j,t}}{\partial p^*_{i,j,t}}. \tag{3}$$

A first hurdle to bring the last equation to the data is that marginal costs are generally unobservable. However, under fairly general assumptions AIK show that marginal costs can be written as the sum of all variable input prices weighted by their respective shares in total variable costs at the firm level, plus a product-specific cost component. Therefore, with information on the cost shares of specific inputs, ϕ^c_{jt}, (e.g. from firms' balance sheets) and on their prices (Δp^c_{jt}), also controlling for competitors' prices, Equation (3) can be implemented in a linear regression framework to estimate the structural pass-through coefficient $\frac{1}{1+\Gamma}$.[13]

3.2 Cost pass-through under price stickiness: Extensive and intensive margins

Price adjustment to cost shocks may be lumpy and not instantaneous for a variety of reasons. Regarding empirical specifications of Equation 3, infrequent and lumpy price adjustment raises the following two observations. First, including unchanged prices among the dependent variable $\Delta p^*_{i,j,t}$ will bias the estimates downward. This bias is present under both time-dependent and state-dependent price setting (e.g. Berger and Vavra (2019) formally show that the bias is proportional to the frequency of adjustment). To be clear, zero price changes are crucial to understand aggregate inflation dynamics, but it is equally crucial to precisely estimate how much firms change their prices conditional on adjustment. This intensive margin is central to shed light on the role of real rigidities in price adjustment separately from that of nominal rigidities. Therefore, a typical solution in the empirical literature is to run pass-through regressions conditioning on non-zero price changes.

However, and this is the second observation, even conditioning on non-zero price changes in

[12] When the demand for goods produced by multiproduct firms is not separable and has a different elasticity within the firm than across firms, the good-specific markup becomes a function of the sensitivity of the firm-specific demand for other goods. Therefore, the markup cannot be easily expressed as simply a function of the prices of competitors of the same good as it can be affected by competitors' prices in all these other markets.

[13] Nevertheless in computing $\Delta\overline{p}_{-j,t}$ we cannot easily measure prices of foreign competitors (both for domestic prices and for export prices), so that the estimated $\frac{1}{1+\Gamma}$ in response to import price shocks may also reflect to some extent the elasticity of foreign competitors' prices to Danish imported inputs, for instance due to common suppliers in third countries.

general may not allow recovering the structural pass-through coefficients, in particular under state-dependent pricing.[14] In this case, the above pass-through regression (3) is biased by endogenous selection into optimally "resetting" prices. Selection induces a correlation between the observed cost shock, and any other unobserved good-level idiosyncratic shock. To wit: in standard menu cost models, the price of a good receiving large (unobservable) idiosyncratic shocks of the same sign as the (observable) cost shock of interest is more likely to be adjusted, other things equal. This selection bias is likely to be present at any horizon $t+h$ at which the probability that the price may not change is non-negligible, making OLS estimates potentially biased. The bias could be present even in response to small observable shocks.

The link between selection and state-dependence has been documented numerically by Costain and Nakov (2011) for monetary policy shocks, showing that the higher the degree of state dependence, the larger the average response of firms to these shocks relative to a purely time-dependent (Calvo) model. Here, using the analytical methods recently developed by Alvarez and Lippi (2021) to solve for a broad class of state-dependent models with random walk idiosyncratic cost shocks, we illustrate analytically the link between selection bias and state-dependence for the case of the response to a (small) nominal cost shock common to all firms. The (single-good) class of models in Alvarez and Lippi (2021) flexibly encompasses both the menu cost model of Golosov and Lucas (2007) and the purely time-dependent Calvo model (similarly to Costain and Nakov (2011)). While in the former framework firms decide to change prices endogenously, in the latter the probability of changing prices is determined by an exogenous parameter (denoted by ζ below). Specifically, Alvarez and Lippi (2021) shows that in response to a small permanent nominal cost shock δ at $t_0 = 0$, in this class of models the cumulated aggregate price change (including zero and non-zero

[14] Optimal price adjustment under time-dependent pricing is different from the case of flexible prices in response to the same cost shocks. For instance, assuming a constant markup for simplicity, the optimal flexible price change is always as follows:

$$\Delta p^*_{jt} = \Delta mc_{jt};$$

but in the time-dependent Calvo model this coincides with the optimal *reset* price only when cost shocks are close to a random walk. As shown by Gagnon (2009), in a stationary equilibrium with zero inflation the optimal (log) reset price p^*_{jt} in the Calvo model with idiosyncratic cost shocks is given by

$$p^*_{jt} = const + (1-\beta\zeta)\ln\sum_{s=0}^{\infty}(\beta\zeta)^s \exp\left[\rho_A^s \widehat{C}_{jt+s} + \frac{1-\rho_A^{2s}}{1-\rho_A^2}\sigma_u^2\right],$$

where $1-\zeta$ is the exogenous probability of adjusting prices, and idiosyncratic cost shocks $\widehat{C}_{jt}$ are assumed log-normal as follows:

$$\begin{aligned}
\widehat{C}_{jt} &= \ln\left(C_{jt}/\overline{C}\right)\\
\ln C_{jt} &= (1-\rho_A)\ln\overline{C} + \rho_A \ln C_{jt-1} + u_t\\
u_t &\sim N\left(0,\sigma_u^2\right) => \widehat{C}_{jt}|\widehat{C}_{jt-1} \sim N\left(\rho_A\widehat{C}_{jt-1},\sigma_u^2\right), \widehat{C}_{jt} \sim N\left(0, \frac{\sigma_u^2}{1-\rho_A^2}\right).
\end{aligned}$$

Therefore, the reset price is the same as under flexible prices only when $\rho_A \to 1$, namely shocks are close to a random walk.

changes) at $t_0 + t$, $P(t,\delta)$, could be approximated up to first-order as follows:[15]

$$P(t,\delta) = \delta \left\{ 1 - \sum_{j=1}^{\infty} e^{-\zeta\left[1+\frac{(2\cdot j\pi)^2}{8\phi}\right]\cdot t} \left[\frac{2}{1+\frac{(2\cdot j\pi)^2}{8\phi}} \frac{1-\cosh\left(2\sqrt{\phi}\right)(-1)^{2\cdot j/2}}{1-\cosh\left(2\sqrt{\phi}\right)} \right] \right\}. \tag{4}$$

In the notation in Alvarez and Lippi (2021) the parameter $\phi \in (0,\infty)$ determines how close the model is to Golosov-Lucas ($\phi \to 0$) or to Calvo ($\phi \to \infty$), with intermediate values denoting an intermediate degree of state-dependence. The variable $P(t,\delta)$ represents the aggregate price level response to the shock δ averaging across all zero and non-zero price changes. Since the term in the curly brackets is less than 1, the aggregate price response falls short of the shock δ over all finite horizons, because of price stickiness resulting in some prices remaining unchanged. In a regression framework where (different realizations of) the shock δ could be observed, $P(t,\delta)$ would correspond in population to OLS estimates where individual zero and non-zero price changes between t_0 and $t_0 + t$ are regressed over (different realizations of) δ.

What about adjustment conditional on resetting prices and selection? The analytical methods in Alvarez and Lippi (2019) allow a sharp characterization in this case. Defining with $S(t,\delta)$ the probability of survival of an unchanged price (= fraction of unchanged prices as of t after shock δ in t_0, we can compute an approximation to cumulated non-zero price changes between t_0 and t as the ratio $\dfrac{P(t,\delta)}{1-S(t,\delta)}$. Clearly in the Calvo model with exogenous probability of changing prices $1-\zeta$ in each period, the survival probability is $S(t) = e^{-\zeta t}$, independent of δ, while the average price response including zero price changes is given by the shock δ times $1 - S(t)$, the fraction of prices that are changing between t_0 and t:

$$P(t,\delta) = \delta \cdot \left(1 - e^{-\zeta t}\right);$$

therefore we have the following:

$$Calvo: \frac{P(t,\delta)}{1-S(t)} = \delta.$$

Intuitively, averaging across exogenous non-zero price changes exactly retrieves, with no selection bias, the average marginal adjustment to the shock in reset prices, in this case equal at each horizon to the permanent cost shock δ . Actual price changes reflect both unobservable idiosyncratic shocks and δ, but the former are just a random sample from their distribution across firms and thus wash out in the cross section.

In the Golosov-Lucas model (with $\phi \to 0$) too, Alvarez and Lippi (2021) shows that $S(t,\delta) = S(t)$, i.e. the survival function is also independent of δ up to first order, i.e. when the latter is

[15]The paper looks at a random-walk monetary policy shock which permanently increases marginal costs by δ.

sufficiently small. Hence non-zero cumulated price changes can be approximated as follows:

$$GL: \frac{P(t,\delta)}{1-S(t)} = \delta \frac{1-\sum_{j=0}^{\infty} \frac{32}{((2+4j)\pi)^2} e^{-N\frac{((2+4j)\pi)^2}{8}\cdot t}}{1+\sum_{j=1}^{\infty} e^{-N\frac{((2j-1)\pi)^2}{8}\cdot t}\left[\frac{2}{(2j-1)\pi}\left(\cos\left((2j-1)\pi\right)-1\right)\sin\left(j\cdot\frac{\pi}{2}\right)\right]} \approx \delta \frac{1-\frac{32}{(2\pi)^2}e^{-N\frac{(2\pi)^2}{8}\cdot t}}{1-\frac{4}{\pi}e^{-N\frac{\pi^2}{8}\cdot t}},$$

where N denotes the number of adjustment per unit of time (equal to ζ in the Calvo model), and for simplicity in the last expression on the right-hand side we have focused on the first (dominant) non-zero terms in the summations. Clearly, the ratio in the last expression on the right-hand side is larger than 1, since[16]

$$e^{-N\frac{\pi^2}{8}\cdot t} > \frac{2}{\pi}e^{-N\frac{(2\pi)^2}{8}\cdot t}, \forall t > 0.$$

This implies that averaging across non-zero state-dependent price changes overestimates the correct marginal adjustment in reset prices to the cost shock δ because of endogenous selection into price adjustment, for all (finite) horizons t. Remarkably, this is true even though the probability of changing prices (as determined by the survival function $S(t,\delta)$) is unaffected up to first order by the small shock δ.

Finally, we can approximate non-zero price changes for the intermediate case $\phi \in (0,\infty)$, obtaining (again focusing on the first dominant term) the following:

$$\frac{P(t,\delta)}{1-S(t)} = \delta \frac{1-\sum_{j=1}^{\infty} e^{-\zeta\left[1+\frac{(2\cdot j\pi)^2}{8\phi}\right]\cdot t}\left[\frac{2}{1+\frac{(2\cdot j\pi)^2}{8\phi}}\frac{1-\cosh\left(2\sqrt{\phi}\right)(-1)^{2\cdot j/2}}{1-\cosh\left(2\sqrt{\phi}\right)}\right]}{1+\sum_{j=1}^{\infty} e^{-\zeta\left[1+\frac{((2j-1)\pi)^2}{8\phi}\right]\cdot t}\left[\frac{2}{(2j-1)\pi}\left(\cos\left((2j-1)\pi\right)-1\right)\sin\left(j\frac{\pi}{2}\right)\right]} \approx \delta \frac{1-\frac{2}{1+\frac{(2\pi)^2}{8\phi}}\frac{1+\cosh\left(2\sqrt{\phi}\right)}{1-\cosh\left(2\sqrt{\phi}\right)}e^{-\zeta\left[1+\frac{\pi^2}{2\phi}\right]\cdot t}}{1-\frac{4}{\pi}e^{-\zeta\left[1+\frac{\pi^2}{8\phi}\right]\cdot t}}.$$

It is relatively straightforward to show that for $\phi \in (0,\infty)$ the ratio on the right hand side is larger than 1 but decreasing in ϕ, the degree of time dependence.[17] This implies that the selection bias is decreasing in the degree of time dependence, vanishing in the Calvo model as shown above.[18]

The bottom line is that it is important to take selection into changing prices and its likelihood

[16]Focusing on the first dominant term implies that the approximation is not as precise for t very close to zero, since at $t=0$ the denominator is negative, see Alvarez and Lippi (2019). Specifically, the approximation holds for $N \cdot t$ such that the denominator in the expression is positive, namely:

$$1-\frac{4}{\pi}e^{-N\frac{\pi^2}{8}\cdot t} > 0 <=> N\cdot t > 8\frac{\ln(4/\pi)}{\pi^2} \simeq 0.196.$$

[17]Again, when the denominator on the right-hand side is positive for t sufficiently large, namely:

$$\zeta \cdot t > \frac{\ln(4/\pi)}{\left[1+\frac{\pi^2}{8\phi}\right]}.$$

[18]Formally:

$$\frac{1}{\pi}e^{-\zeta\left[1+\frac{\pi^2}{8\phi}\right]\cdot t} > \frac{4\phi}{2\phi+\pi^2}\frac{1+\cosh\left(2\sqrt{\phi}\right)}{1-\cosh\left(2\sqrt{\phi}\right)}e^{-\zeta\left[1+\frac{\pi^2}{2\phi}\right]\cdot t}, \forall t \geq 0.$$

into account when estimating cost pass-through at different time horizons, particularly in the short-run. Moreover, as we discuss below, accounting for selection bias is crucial to provide direct evidence on the quantitative significance of state-dependence in shaping price adjustment.

3.3 Selection-bias corrected estimation

In order to estimate cost pass-through taking into account state-dependence and selection into changing prices, we propose the following two-step procedure, drawing from the selection bias correction approach by Bourguignon et al. (2007).[19] Specifically, in the first step we model endogenous selection into price adjustment as a multinomial logit, while in the linear projections in the second step we include a "bias correction" based on the first step.

Consider the following local projection model of joint extensive and intensive margin of price adjustment over horizons $h = 0, ..., H$:

$$\begin{aligned} r^*_{ij,m,t+h} &= \gamma^h_m Z_{ijt} + \eta_{ij,m,t+h}, \qquad m = -1, 0, 1 \\ p_{ij,t+h} - p_{ij,t-1} &= \beta^h X_{ijt} + u_{ij,t+h}, \qquad m \neq 0 \end{aligned} \tag{5}$$

where r^* is the (profit) outcome of a categorial variable m taking (without loss of generality) the value 1 if the price increases between periods t and $t+h$, i.e. $p_{ij,t+h} - p_{ij,t-1} > 0$, -1 if the price decreases, and 0 otherwise. Maximizing firms choose a positive cumulated change in reset prices, $p_{ij,t+h} - p_{ij,t-1}$, if $r^*_{ij,1,t+h} > \max(r^*_{ij,m,t+h}), \forall\, m$. Observe that (5) assumes that coefficients γ^h_m and β^h are specific across outcomes m and horizons h. In particular, this flexible specification implies that explanatory variables Z_{ijt} can have asymmetric effects at any horizon h on the probability of (cumulated) price hikes or cuts, so that outcomes m are not ordered.

Under the standard assumption that $\eta_{ij,m,t+h}$ is (cross-sectionally) independently and identically Gumbel distributed, this leads to a multinomial logit model for each horizon h (McFadden, 1973, Dubin and McFadden, 1984):

$$\Pr\left(m_{ij,t+h} = 1, 0, -1 \,\middle|\, Z_{ijt}\right) = \Phi\left(\gamma^h_m Z_{ijt}\right) = \frac{e^{\gamma^h_m Z_{ijt}}}{1 + \sum_m e^{\gamma^h_m Z_{ijt}}}. \tag{6}$$

Nevertheless, we also test the robustness of our results when the extensive margin is modelled as

Observe that for $\phi \to \infty$ the expressions do not exactly converge to those for the Calvo model, for reasons explained in Alvarez and Lippi (2021). Morever, the results here on selection bias obviously are tightly related to the discussion on selection and monetary non-neutrality in Section 5 of that paper.

[19] The logic is similar to a Heckman (1979) bias-correction with more than two categorical outcomes in the first step. The literature has often relied on Tobit Type II to accomodate discreteness in price changes, but given the binomial restriction of its outcome variable, the model needs to be estimated twice to account for asymmetries in the probability of price increases and decreases (Berardi et al., 2015). We argue that our multinomial approach is better suited for this purpose, as the selection will be a byproduct of estimating only one intensive-margin equation.

a multinomial probit, in which it is assumed that $\eta_{ij,m,t+h}$ is normally distributed with abitrary (cross-sectional) correlation across outcomes m.[20]

The second equation in (5) estimates the intensive margin of adjustment of reset prices conditional on observing outcome $m \neq 0$ in the first step. We proceed as follows. First, as it is standard in the literature, in order to enhance identification and minimize multicollinearity, we include among the second step regressors X_{ijt} only a subset of the first step regressors Z_{ijt} — whereas the choice of the enlarged set of regressors Z_{ijt} is guided by economic theory (see Section 4 for the exact implementation). Second, consistent estimation of the cost pass-through elasticities β^h requires additional restrictions because the error term $u_{ij,t+h}$ might be dependent on $\eta_{ij,m,t+h}$, introducing correlation between explanatory variables and the disturbance term in the second equation in (5), as for instance implied by state-dependent pricing. These additional restrictions take the form of linearity assumptions on the dependence between the model residuals $(u_{ij,t+h}, \eta_{ij,m,t+h})$. Specifically, we follow variant 2 of the Dubin-McFadden approach in Bourguignon et al. (2007), which assumes that the conditional expectation of the error terms in the (linear projection) second step is a linear function of know convolutions of the errors in the first step, yielding the following second step estimation equation for cumulated changes in reset prices between t and $t + h$:

$$p_{ij,t+h} - p_{ij,t-1} = \beta^h X_{ijt} + \underbrace{\lambda^h_{m^*}\mu(\mathrm{Pr}_{h,m^*}) + \sum_{m \neq m^*} \lambda^h_m \left(\mu(\mathrm{Pr}_{h,m}) \frac{\mathrm{Pr}_{h,m}}{(\mathrm{Pr}_{h,m} - 1)} \right)}_{\text{selection bias correction}}, \quad m^* \neq 0; \tag{7}$$

the functions $\mu(\cdot)$ are integrals over the individual probabilities from the multinomial first step, to be computed numerically. Note that we allow the reset price elasticity β^h to vary across horizons but exclude unchanged prices from the second step altogether to minimize any residual bias. The aim of the selection bias correction term in (7) is to correct the bias induced by endogenous selection into resetting prices.

3.4 A back-of-the-envelope decomposition into "time-dependent" and "state-dependent" price adjustment

The two-step approach discussed above yields estimates of how observable cost shocks (as included in the regressors Z_{ijt} and X_{ijt}) affect both the probability of changing prices (via the multinomial logit coefficients γ^h_m), and the adjustment in reset prices in response to these shocks, corrected for selection bias (via the second step coefficients β^h). Therefore, a test of whether the coefficients

[20]The selection bias correction for this case is derived in Appendix E. Estimable functions for selection-bias corrected multionomial logit and probit are available from the authors.

$\lambda_m^h \neq 0$ in the second step equation (7) provides evidence on the statistical significance of selection bias. Nevertheless, from a modeling perspective it is also informative to document the economic relevance of the different channels of price adjustment in response to cost shocks. In order to do that, we rely on the inflation decomposition proposed by Caballero and Engel (2007), adapting it to price changes *conditional* on a cost shock.

Specifically, taking as a measure of inflation dynamics after a shock the (estimated) average price response between t and $t+h$ (including zero and non zero price changes), $\widehat{p}_{t+h} - \widehat{p}_{t-1}$, we propose to decompose the latter as follows:

$$\widehat{p}_{t+h} - \widehat{p}_{t-1} = \underbrace{\overline{\lambda}_h \cdot (\widehat{p}^*_{t+h} - \widehat{p}^*_{t-1})}_{\text{time-dependent adj.}} + \underbrace{\Big[(\widehat{p}_{t+h} - \widehat{p}_{t-1}) - \overline{\lambda}_h \cdot (\widehat{p}^*_{t+h} - \widehat{p}^*_{t-1})\Big]}_{\text{state-dependent adj.}}, \tag{8}$$

where the first right-hand side term $\overline{\lambda}_h(\widehat{p}^*_{t+h} - \widehat{p}^*_{t-1})$ represents the fraction of adjustment consistent with time-dependence, given by the unconditonal probability of changing prices over h periods, $\overline{\lambda}_h$, times the (estimated) desired marginal adjustment across reset prices in response to the observable shock, $(\widehat{p}^*_{t+h} - \widehat{p}^*_{t-1})$. Hence, this "time-dependent" adjustment can be thought of as the response of the aggregate price level (averaged over "reset" prices and unchanged prices) that would be consistent with a purely time-dependent model — whereas the probability of price changes is unaffected by shocks, and state-dependence and selection do not impact reset prices (so that e.g. any idiosyncratic factor washes out in the cross-section of reset prices).

Conversely, the second (residual) term on the right-hand side of (8) represents the contribution to the aggregate inflation response above and beyond what a simple time-dependent model can account for. In the spirit of the decomposition by Caballero and Engel (2007), the second term must thus broadly reflect "state-dependent" adjustment. Moreover, as shown by Costain and Nakov (2011), in state-dependent models the latter is the sum of the contributions of selection proper, and of shifts in the probability of price changes in response to (observable) cost shocks.[21]

We make this decomposition operational by observing that for a given cost shock (included in the regressors X_{ijt}), an estimate of the term $(\widehat{p}^*_{t+h} - \widehat{p}^*_{t-1})$ is exactly what is provided by our selection-corrected pass-through elasticities (β^h), while an estimate of the average price response to the shock, $\widehat{p}_{t+h} - \widehat{p}_{t-1}$, is provided by an OLS regression of zero and non-zero cumulated price changes also over the same cost shocks (in X_{ijt}). Furthermore, we can gauge separately the contribution of selection in the narrow sense as the difference between the OLS coefficients of a regression of

[21] Caballero and Engel (2007) dubbed the first and second component, intensive and extensive margin, respectively. However, as pointed out by Costain and Nakov (2011), the second component also includes the contribution by selection to average adjustment of reset prices in state-dependent models, not only the effect of shocks on the probability of changing prices (which many contributions in the literature dub the extensive margin instead). Therefore, we prefer to use our terminology for the decomposition in (8).

non-zero cumulated price changes over X_{ijt}, minus the selection-corrected pass-through elasticities β^h. In the next Section 4, we will present the results of this decomposition by using estimates from Danish producer prices for energy and import cost shocks by applying the two-step approach described in this section.

4 Evidence on extensive and intensive margins of price adjustment

In this Section we present evidence on state-dependence and the extensive margin and intensive margin of price adjustment. We start by describing variables we use in the vectors Z_{ijt} and X_{ijt}. First and foremost, we include the cost shocks, given by $\phi^E_{jt-1}\Delta\widehat{p}^E_t$, the firm-level energy share times the Baumeister and Hamilton (2019) oil supply shock (scaled to represent a 1% change in the Danish energy price), and $\phi^M_{jt-1}\Delta p^M_{jt}$, our measure of firm specific marginal import costs, given by the firm-level import share times the average import prices (in Danish kroner) reported by the firm — both shares are lagged and computed over the previous calendar year. Clearly the first shock is likely to have a larger common component across firms than the second one.

We also include the following regressors in both Z_{ijt} and X_{ijt}: controls for firm level costs, namely the change in domestic and imported purchases over the last 3 months and the average hourly wage times the labor cost share; we also control for the 3-month change in firm-level sales to capture firm specific demand factors. To further control for more aggregate factors affecting firm-level costs we also include the monthly CPI inflation rate and the change in the Danish nominal effective (trade-weighted) exchange rate (NEER); we also include the average price change of competitors at the good-level, $\overline{p}_{i,-j,t}$. This can be motivated not only as proxying for common shocks but also on the basis of firm-level strategic complementarities, as discussed above. Specifically, we define competitors to be firms that sell products in the same 2-digit category of the Harmonized System in the same month, resulting in 74 such product sectors (with an average number of competing firms equal to 42). We will refer to the geometric average of all known firms prices in the same product sector as the price change of competitors (see the appendix for details).

Moreover, both Z and X include sector fixed effects and good-level dummies for exports, temporary sales, product replacements which we identify as changes in the base price at resampling as well as breaks (see Appendix A for details). We also control for the size of the firm by including the log number of employees and for the number of products. Finally, we include month fixed effects (dummies) to control for seasonal time-dependence in the frequency of price changes and seasonality in the size of price changes (though the latter is much less pronounced).[22]

To facilitate identification of the second-step coefficients non-parametrically – as is customary in the literature on selection bias – we use exclusion restrictions, by including some variables only in the multinomial logit estimation step, while excluding them from the second linear projection step. Guided by theoretical considerations from the literature on state-dependent price setting in multiproduct firms, we include among the first-step covariates Z_{ijt} the following variables, which are then excluded from the linear projections in the second step. First, we use the fact that most firms in our sample sell many products whose prices we observe (see Section 2.1.1). In line with Alvarez and Lippi (2014) and Bhattarai and Schoenle (2014), we use the fractions of positive and negative price changes within the same firm, excluding the price change of the good we are trying to explain. Note that these fractions may be expected to have different influences on the likelihood of increasing or decreasing prices, and our approach allows for that. Second, we include the fraction of positive and negative price changes in the same industry at the 2-digit NACE sector (excluding firm j), excluding the i-th good price. Finally, we also include the age of the price (months since the last price change) and the standard deviation of (all) price changes in the firm in the last 5 years; these variables are related to the likelihood of resetting prices in both time-dependent and state-dependent models.

In the rest of this section we first document the properties of the two shocks we consider, also presenting impulse responses of firm-level costs. Then we show sequentially results for the first step (extensive margin) and the second estimation step (intensive margin).

4.1 Cost shocks

In the following Figures 3 and 4 we show the impulse responses of the cost variables itself, as well as firm-level total cost measures, to the energy and import marginal cost shocks. In these figures (and in the following ones) the light grey areas indicate 95% HAC robust confidence bands, where standard errors are clustered at the firm level.

4.1.1 Import cost shock

Figure 3c shows the response of firm-level import costs (the previous year firm-level import share times the actual cumulated firm-specific average price of imports $p^M_{j,t+h} - p^M_{j,t-1}$), and the right-hand side panel shows the response of firm-level total variable costs (domestic and imported intermediates plus the wage bill). By construction the response of import costs is normalized to 1% on impact,

[22] Given their computational complexity in the multinomial logit step we do not include firm-level fixed effects; we plan to explore their role in future revisions of the paper.

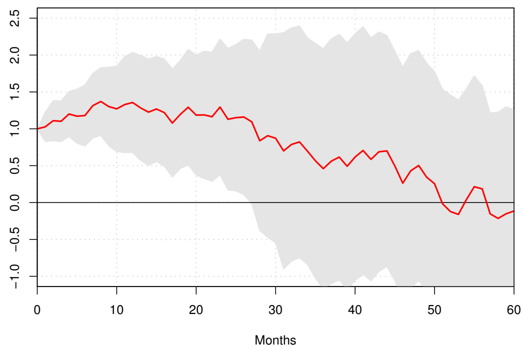
implying an equivalent increase in marginal costs (up to first order). Firm-level marginal import costs are affected very persistently but they revert to baseline over time: point estimates are around 0.5% after 4 years and become statistically insignificant at the 5 year horizon. Firm-level total variable costs also increase persistently and in a statistically significant fashion, reflecting not only the increase in cost of import purchases but, to a smaller extent, also total costs of domestic inputs. On the basis of this mean-reverting cost dynamics, we would expect that firms changing their prices would do so by less than one-to-one with the initial 1% increase in marginal costs. Moreover, actual price changes should also show a tendency to decline over time in the wake of the shock, as import costs revert to baseline.

4.1.2 Energy cost shock

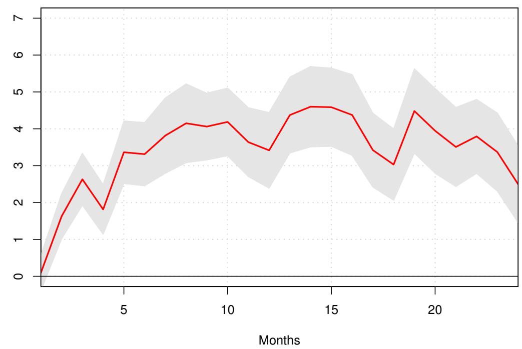
In the case of the (scaled) oil supply shocks with which we proxy energy cost shocks, the left-hand side panel of Figure 3a shows the response of the previous year firm-level energy share times the actual cumulated price of energy in Denmark, $p^E_{t+h} - p^E_{t-1}$; the right-hand side panel again shows the response of total variable costs (i.e. wages plus domestic and imported intermediates). While the cumulated oil supply shocks follow a random walk by construction as they are i.i.d., the response of our proxy for the marginal cost of energy at the firm level starts declining after 2 years and quickly becomes insignificantly different from zero (again the impact response is normalized to 1%). However, looking at the response of total variable costs in the right-hand side graph, it is clear that the shock after a few months persistently affects also all other firm's variable costs, much more than the import cost shock. While in line with the prior that this shock has much larger economy-wide effects than the import cost shock, the pervasive response of total costs is important to keep in mind when interpreting conditional price adjustment to the energy cost shock, since it could affect the elasticity and dynamics of the price response to the shock. For instance, finding a larger price adjustment elasticity to this shock than to the import shock could reflect a response to all cost components. Moreover, it is also likely that firms in different positions in the supply chain will be affected by the shock at different times, depending on the timing of the reaction of their suppliers. As we show below, these "pipeline" pressures are an important feature of the propagation of (oil-supply driven) energy shocks to firms' prices and inflation.

Figure 3: Energy cost shock

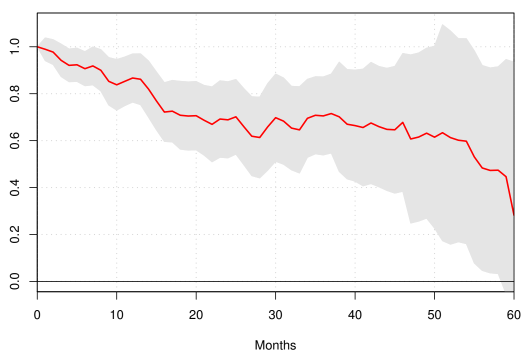
(a) Shock



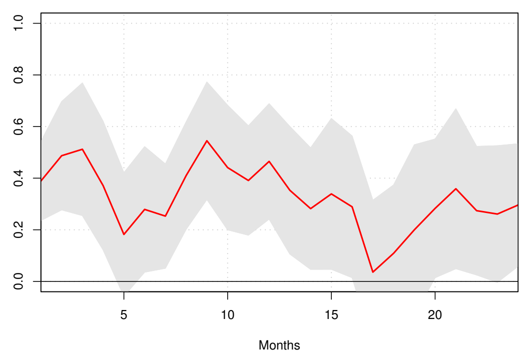
(b) Total variable cost



(c) Shock



(d) Total variable cost



Note: Panels (a): Estimated coefficients of firm-level regressions of leads of the cost share variable ϕ interacted with the input cost changes on the contemporaneous shift-share shock. Panels (b): of regressions of cumulative changes of total variable cost (defined as the sum of labor cost from payrolls registers and domestic and imported inputs from VAT data) on the same regressors. 95% confidence intervals in grey. Standard errors are clustered by firm, except for Panel 3a, where we cluster standard errors only by time.

Figure 4: Import cost shock

4.2 Evidence on state-dependence and synchronization of price changes

As we discussed in Section 3.3, in the first stage we estimate the following multinomial logit model:

$$\Pr\left(m_{ij,t+h} = 1, 0, -1 \,\middle|\, Z_{ijt}\right) = \Phi\left(\gamma_m^h Z_{ijt}\right) = \frac{e^{\gamma_m^h Z_{ijt}}}{1 + \sum_m e^{\gamma_m^h Z_{ijt}}},$$

where $m_{i,j,t+k}$ is an indicator variable for positive, zero, or negative price changes of good i produced by firm j, cumulated between time t and $t+h$, with 0 as the base (no price change) category.

To preview our results, the following stand out. First, there is substantial synchronization of price changes within a firm, consistent with some degree of complementarities in the cost of changing prices. Specifically, we find that, other things equal, the likelihood of an individual price cut (hike) rises with the number of positive (negative) changes in the other prices within a firm, consistent with common costs of changing prices. Second, there is significantly more synchronization of individual adjustment decisions at the firm level relative to the industry. Third, we find evidence for state-dependent pricing in response not only to our cost shocks, but also to other variables such changes in competitors' prices and aggregate inflation.

Table 2 shows the results of the multinomial logit model for the horizon $h = 0$, where the top panel reports results for price hikes and the bottom panel for price cuts. We report marginal effects on the change in the probability of adjustment, given 1% changes around the mean of Z_{ij} — only for firm and industry shares of price changes we consider one-standard-deviation changes around the mean of Z_{ijt}, implying around a 1/3 fraction of other prices changing. We present results for all firms and by splitting them in two groups according to the average number of their product (no more than 5 goods, and more than 5 goods, respectively).

A key results is the presence of *imperfect* synchronization within the firm. Specifically, the probability of raising prices significantly increases with *both* the fraction of positive and negative price changes. These results are strongly consistent with synchronization in price changes because of both firm-level shocks to marginal costs, for the fraction of same-signed price changes, and common costs of changing prices within the firm, for the fraction of opposite-signed price changes.

Conversely, we find quantitatively smaller evidence of synchronization at the industry level. The probability of a positive (negative) price change increases with the fraction of positive (negative) price changes in the same industry, but it is in general much less affected by the fraction of price changes with the opposite sign. The marginal effects are an order of magnitude smaller than those for the within-firm synchronization. This evidence seems consistent with common shocks to marginal costs across firms rather than strategic complementarities, since it is entirely driven by firms with fewer products. Synchronization in the likelihood of price adjustment across firms is thus decreasing with the number of products, in contrast with synchronization within firms.

However, the former effect is almost twice than the latter for price increases. The degree of price synchronization within the firm increases in the number of goods produced by firms, in line with models with some degree of complementarity in the cost of changing prices (see e.g. Bonomo et al. (2019)).[23]

[23] We provide a more direct replication of this result found by Bhattarai and Schoenle (2014) in Appendix D.1.

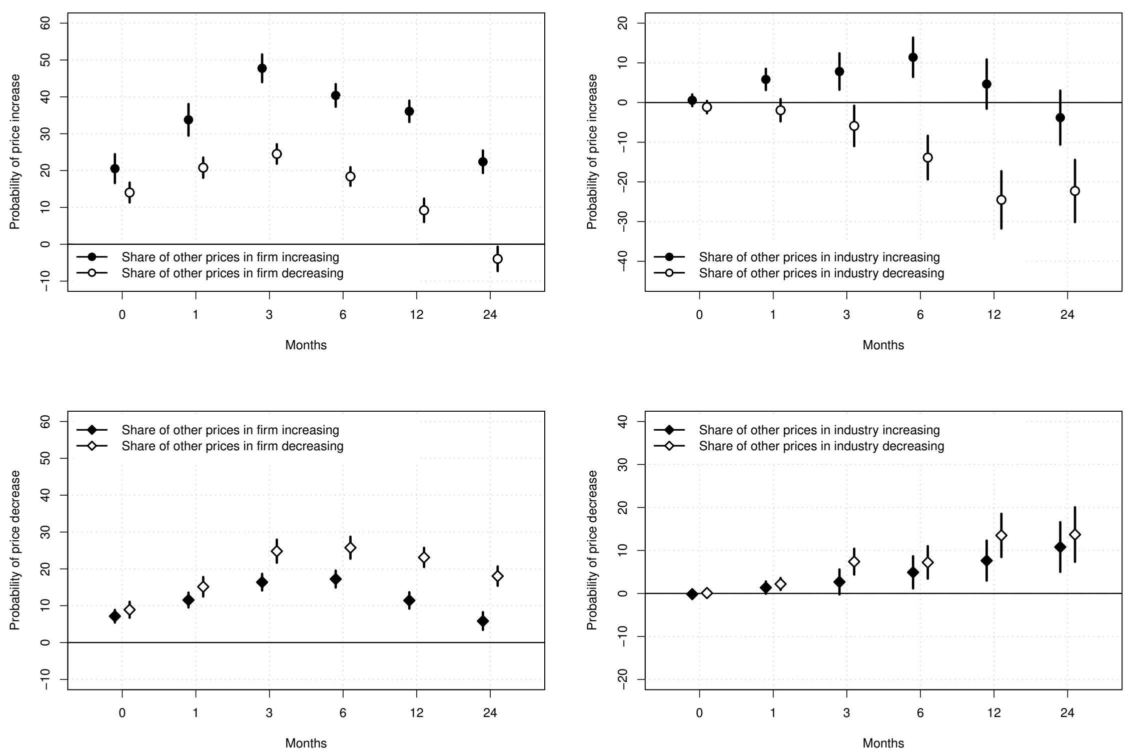
Table 2: Multinomial logit, first stage results

	All	1-5	5+
Marg. effect on probability of price increase			
Fraction of pos. price changes in firm	5.36***	5.02***	5.93***
	(0.19)	(0.20)	(0.29)
Fraction of neg. price changes in firm	2.98***	2.91***	3.11***
	(0.14)	(0.15)	(0.20)
Fraction of pos. price changes in industry	0.05	0.28**	−0.01
	(0.12)	(0.10)	(0.10)
Fraction of neg. price changes in industry	−0.09	0.08	−0.10
	(0.06)	(0.14)	(0.05)
Age of current price	−0.22***	−0.17***	−0.25***
	(0.02)	(0.02)	(0.02)
Standard deviation of recent price changes	58.87***	93.19***	15.09
	(10.49)	(10.96)	(8.97)
Avg. price change in industry, excl. firm	0.05*	0.05	0.01
	(0.02)	(0.02)	(0.03)
Energy cost shock	−0.13	−0.21	−0.19
	(0.09)	(0.13)	(0.11)
Import cost shock	0.14***	0.27***	0.08*
	(0.04)	(0.07)	(0.03)
CPI, log difference	0.17	0.14	0.25
	(0.20)	(0.31)	(0.25)
Marg. effect on probability of price decrease			
Fraction of pos. price changes in firm	1.83***	1.77***	2.02***
	(0.16)	(0.15)	(0.23)
Fraction of neg. price changes in firm	1.86***	1.71***	2.20***
	(0.17)	(0.15)	(0.27)
Fraction of pos. price changes in industry	−0.02	0.02	−0.04
	(0.05)	(0.05)	(0.05)
Fraction of neg. price changes in industry	0.01	0.21***	−0.02
	(0.05)	(0.06)	(0.03)
Age of current price	−0.22***	−0.18***	−0.25***
	(0.01)	(0.01)	(0.01)
Standard deviation of recent price changes	34.16***	49.85***	12.00*
	(6.48)	(6.66)	(5.57)
Avg. price change in industry, excl. firm	−0.05***	−0.03*	−0.06**
	(0.01)	(0.01)	(0.02)
Energy cost shock	−0.09	−0.13*	−0.03
	(0.05)	(0.06)	(0.08)
Import cost shock	−0.07***	−0.13***	−0.02
	(0.02)	(0.04)	(0.02)
CPI, log difference	−0.31*	−0.77***	−0.12
	(0.14)	(0.18)	(0.15)
N	272,758	105,225	167,533
R2	0.433	0.489	0.412

Significance levels: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Note: Marginal effects (in percentage points) on increasing and decreasing relative to not changing the price. The variables of within firm and industry synchronization show the effect of a one standard deviation change of the regressor around its mean; the coefficients on current price age and recent price change volatility are for unit changes (in months), and the remaining show the effect of a 1% increase in the marginal cost, competitors' prices or the CPI, respectively. Standard errors in parentheses. Further controls as described at the top of Section 4.

Figure 5: Marginal effects of price synchronization

(a) within firm (b) across firms within industry

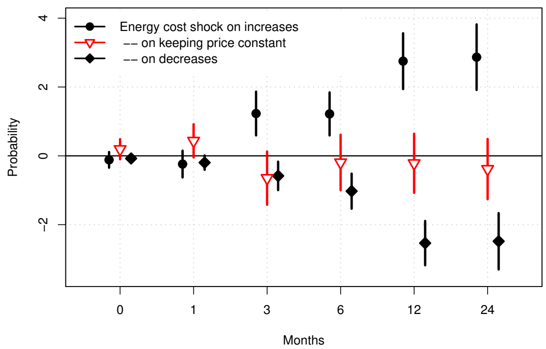


Note: Marginal effects of a unit increase in the regressors on the probability of an increase (top) or decrease (bottom) in the price after h months. The regressors include the share of positive and negative price changes at the firm level, excluding the good (left panels) and the equivalent share at the industry level excluding the firm (right). Further controls as in Table 2.

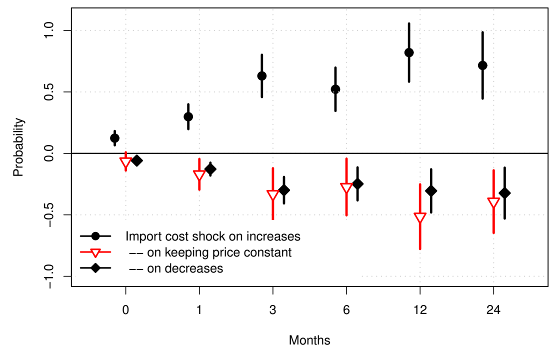
Figure 5 looks at the effects of synchronization over time. Specifically, the figure reports the marginal effects on cumulated probabilities for selected horizons h of setting to 1 the share of positive (black) and negative (white) price changes within the firm, in the case of price hikes (first row) and cuts (second row). The figure shows that the effects of *current* synchronization persist over time, affecting the likelihood of cumulated price changes well into the future. The first column of Figure 5 shows that the marginal effects of the within-the-firm shares of positive and negative price changes peak between 3 to 6 months, thus increasing the probability of interrupting an unchanged price spell over these horizons by between 20 and 40 percentage points, and are still significant for $h = 12$. Interestingly, this persistence is in line with the model with multiproduct firms by Bonomo et al. (2019). The marginal effects for the fraction of price changes across firms and within an industry, shown in the second column, display a similarly persistent dynamics, but are quantitatively smaller.

Figure 6: Marginal effects of marginal cost and other state dependence

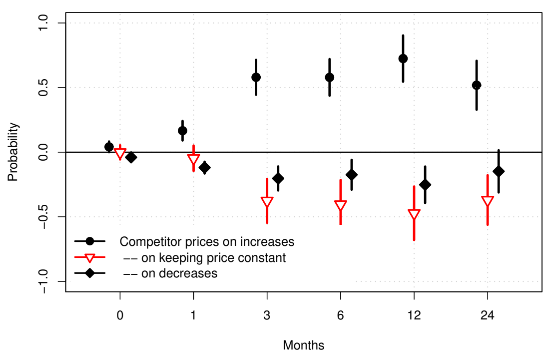
(a) Energy cost shock



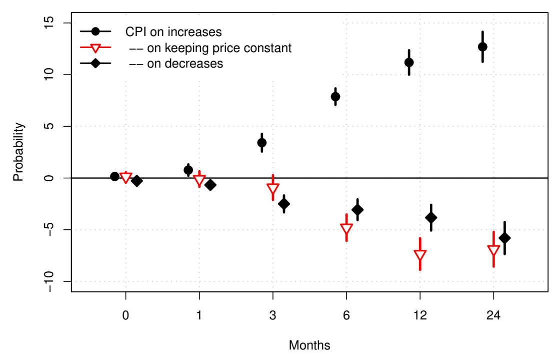
(b) Import cost shock



(c) Competitor prices



(d) Consumer price index



Note: Marginal effects of a unit increase in the following regressors on the probability of an increased/unchanged/decreased price after h months. Further controls as in Table 2.

Our second set of results speaks to a long-debated and important question in macroeconomics, namely the degree of time-dependence or state-dependence in price setting. On the one hand, we find that there is significant time-dependence in the probability of changing prices because of calendar effects. Specifically, the probability of a price increase is significantly larger in January, April, July and October, than in other months, irrespective of the number of goods produced. Interestingly, the seasonal pattern for price decreases instead is not statistically significant.

On the other hand, there is evidence in support of state-dependent pricing. Consistent with models with some fixed costs of changing prices, not only is the probability of price hikes and cuts increasing in its past volatility on impact (as shown in Table 2).[24] Several time series variables also

[24] The age of the price is also significant but its marginal effect is negative; this can reflect the fact that prices which stay unchanged for longer do so because the cost of resetting them is larger — recall that we do not include price-specific fixed effects in our estimation.

significantly and persistently affect the probability of price changes over time. Specifically, a 1% increase (decrease) in energy costs ($\phi^E_{jt-1}\Delta\widehat{p}^E_t$), import costs ($\phi^E_{jt-1}\Delta p^M_{jt}$), the aggregate CPI, the NEER and competitors' prices all significantly raise the likelihood of a price hike (cut), and reduce the probability of a price cut (hike). As shown in Figure 6, which reports these marginal effects over selected horizons for price increases (black dots) decreases (black diamonds) and constant prices (red triangles), they build up over time and are very persistent. CPI changes, shown in the bottom row, have the larger effect, implying that at its peak at $h = 12$ a 1% rise significantly increases the probability of a higher (lower) cumulated price by 11% (4%); as a result, the probability that a price is unchanged 12 month after the 1% CPI increase falls by around 7% in a stastically significant way. The top row of Figure 6 shows that the marginal effects for a 1% rise (fall) in marginal cost due to energy or import prices imply a statistically significant increase in the probability of a price hike (cut) of 3% (2.5%) and 1% (0.5%), respectively. Import cost shocks also have a statistically significant impact on the probability of survival of an unchanged price spell at each horizon, whereas a positive 1% shock reduces this probability by -0.5% after 12 months. Nevertheless,the marginal effects of energy shocks on this survival probability are marginally significant only at $h = 3$. This result on the different degree of state dependence can reflect the fact that energy cost shocks are less volatile than import cost shocks — the respective standard deviations are 0.3% and 1.1%.[25]

4.3 The intensive margin of price adjustment to cost shocks

In this section we report the results of the estimation in the second stage of the dynamic pass-through conditional on price adjustment. We use local projections à la Jordà (2005), where the dependent variable is the cumulated price change of product i of firm j from period t to $t + h$, $p_{i,j,t+h} - p_{ij,t-1}$, conditional on it being non-zero over this time interval. In line with our two-step procedure to take into account endogenous selection, we include "correction" terms from the first stage estimation for each horizon h.

Figure 7 presents three estimates for each horizon h of the price pass-through coefficients of $\phi^E_{jt-1}\Delta\widehat{p}^E_t$ (in panel (a)) and $\phi^M_{jt-1}\Delta p^M_{jt}$ (in panel (b)); In the Figure, the dashed black line shows OLS estimates of pass-through coefficients including zero and non-zero price changes, while the solid black line shows OLS estimates of pass-through coefficients including only non-zero price changes; the red line shows estimates conditional on non-zero price changes corrected for selection bias, for which the light grey areas indicate 95% HAC robust confidence bands.[26]

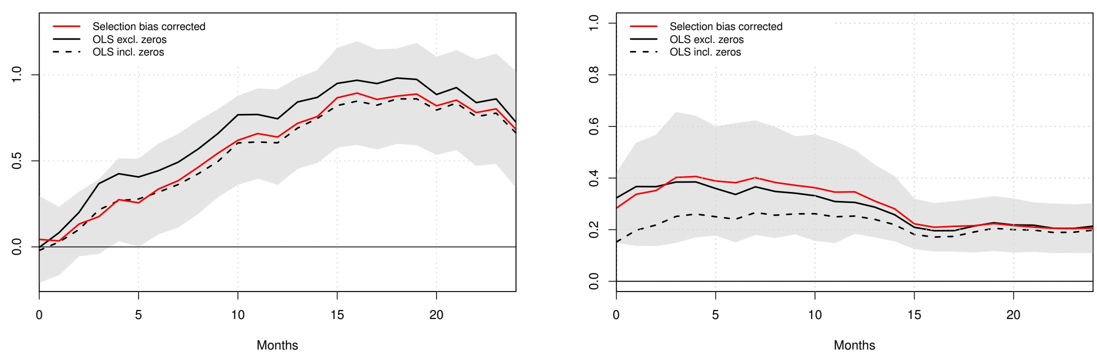
[25] Indeed, under time-dependence, a positive cost shock would increase the probability of price hikes and reduce the probability of price cuts by the same amount, while leaving unchanged the probability of keeping prices constant.

[26] Standard errors are clustered at the firm level and corrected for first stage uncertainty, a process which we explain in detail in Appendix ??.

Figure 7: Cost shock pass-through estimates: Intensive margin

(a) Energy cost shock

(b) Import cost shock



Note: Estimated coefficients of good-level cumulative price changes on two shocks, both of which are constructed as lagged share of the input in total cost and the input price change. The solid red line describes the selection-bias corrected estimation proposed Section 3.3. 95% confidence bands in grey; standard errors are clustered at the firm level and corrected for first-step uncertainty.

The following results stand out. First, despite the similar immediate and persistent increase in both firm-level measures of marginal costs in Figures 3c and 3a, the response of prices is very different across panels (a) and (b). Following the shock to energy costs, prices increase only very gradually, from a small and statistically insignificant level on impact, to around 0.5 after 6 months, and then peak at around 0.8-0.9 after 15 months. Remarkably, this is true of all the different estimates of adjustment in the left-hand side panel, regardless of the exclusion or not of zero price changes (black solid line compared with dashed line), or when correcting for selection bias (red solid line). Conversely, the shock to import costs brings about an immediate and significant increase in prices, with all three estimates broadly stable after 6 months around a value of 0.2-0.3, thus much lower than the estimated elasticity to energy costs. However, (OLS) pass-through estimates over zero and non-zero price changes display a more gradual adjustment in the first 12 months, suggesting a bigger role of price stickiness in short-run adjustment to import costs than energy costs.

Second, conditioning on reset prices, OLS and bias corrected point estimates are very similar over all horizons in the case of both shocks. This is so even though we find that the bias correction terms in the second step are significantly different from zero, as shown in Table 3. The 4th, 5th and 6th columns in the Table, under the heading "Selection", report point estimates of the three selection correction terms (for the probability of decreasing prices, keeping them unchanged or increasing them as shown in Equation 7) for selected horizons h. The correction terms for the probability of price increases and decreases are statistically significant at all horizons, in line with the evidence of state-dependence in first step estimates. Nevertheless, state dependence does not translate into

Table 3: Selection

	$\phi^E_{jt-1}\Delta\widehat{p}^E_t$	$\phi^M_{jt-1}\Delta p^M_{jt}$	$\Delta\overline{p}_{i,-j,t}$	Selection $m=-1$	$m=0$	$m=1$	N	R2
k=0	0.0435	0.2834***	0.1680***	−0.0934***	0.0000	0.0743***	54,653	0.429
k=1	0.0344	0.3372***	0.2471***	−0.1041***	−0.0000	0.0977***	76,149	0.442
k=2	0.1325	0.3517***	0.2554***	−0.1067***	−0.0001	0.1154***	93,309	0.464
k=3	0.1756*	0.4021***	0.2710***	−0.1110***	−0.0007	0.1262***	103,888	0.472
k=4	0.2745**	0.4060***	0.2505***	−0.1218***	−0.0012	0.1288***	113,057	0.481
k=5	0.2562*	0.3885***	0.2417***	−0.1289***	−0.0001	0.1304***	121,267	0.484
k=6	0.3361**	0.3819***	0.2373***	−0.1332***	−0.0002	0.1335***	127,366	0.488
k=9	0.5449***	0.3719***	0.2172***	−0.1577***	−0.0002	0.1256***	141,721	0.493
k=12	0.6383***	0.3465***	0.2204***	−0.1845***	−0.0003	0.1128***	149,625	0.492
k=15	0.8667***	0.2226***	0.1952***	−0.2128***	−0.0005	0.1042***	151,142	0.495
k=18	0.8761***	0.2153***	0.1802***	−0.2317***	−0.0008	0.9975***	150,493	0.499
k=21	0.8529***	0.2098***	0.1427***	−0.2484***	−0.0010	0.0950***	148,591	0.502
k=24	0.6832***	0.2058***	0.1492***	−0.2748***	−0.0014	0.8499***	145,715	0.501

Significance levels: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Note: The first two columns contain pass-through coefficients ($\frac{1}{1+\Gamma}$ in Equation 3), the third column the coefficient on competitor prices in t as the control variable. Columns 4 through 6 contain the coefficient on the selection terms constructed as in Equation 7.

a large bias in OLS estimates of pass-through nor a substantial contribution of selection to price adjustment, as we now show.

In order to better gauge the contribution of state dependence in accounting for the overall price adjustment to our cost shocks, we turn to the decomposition in (8). Table 4 presents the building pieces of the decomposition for selected horizons h. In the first column, the table reports the aggregate price level response to the two shocks, estimated with simple OLS including zero and non-zero price changes; in the 4th column the "time-dependent" adjustment and in the last column the (residual) "state-dependent" adjustment. The column before the last, dubbed "selection", shows the difference between the OLS estimates conditional on resetting prices and the bias corrected estimates, multiplied by the unconditional probability of changing prices shown in the column (3). Clearly, the specific selection contribution to adjustment is in general rather small when compared to time-dependent adjustment.

Figure 8 visualizes the elements of (8), whereas the grey area depicts the 95% confidence band around OLS estimates including unchanged prices (the black solid line) already shown in Figure 7. The fraction of adjustment consistent with time-dependence, the red dashed line, is close to the black solid line and mostly within its 95% confidence band in both panels of Figure 8. This suggests that the bulk of the price response to the shocks we consider would be accounted for

Table 4: Margin decomposition

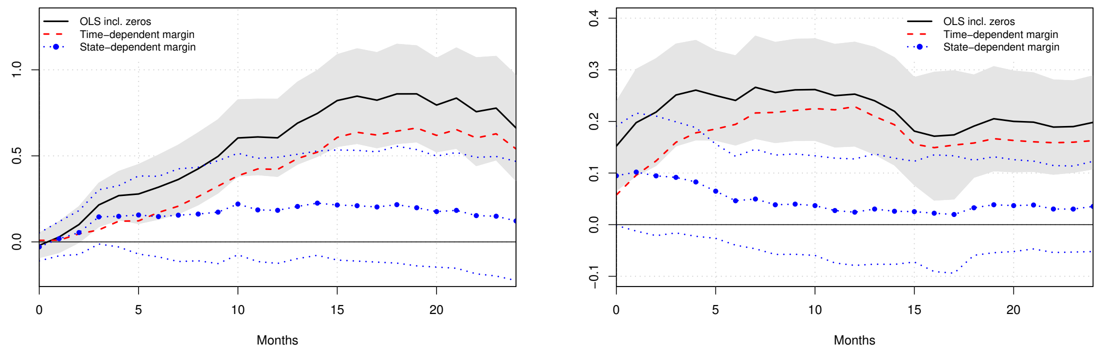
	Total	Intensive margin			Extensive margin		
	$\Delta\widehat{p}_{t+h}$	$\overline{\lambda}_h$	$\Delta\widehat{p}^*_{t+h}$	Total	Observable	Selection	Total
	(1)	(2)	(3)	(2)×(3)	(4)	(5)	(4)+(5)
Energy cost shock							
k=0	−0.0204	0.2030	0.0435	0.0088	−0.0198	−0.0094	−0.0292
k=1	0.0282	0.2842	0.0344	0.0098	0.0046	0.0138	0.0184
k=2	0.1007	0.3512	0.1325	0.0465	0.0304	0.0239	0.0542
k=3	0.2148	0.3968	0.1756	0.0697	0.0690	0.0762	0.1452
k=4	0.2688	0.4381	0.2745	0.1203	0.0825	0.0660	0.1486
k=5	0.2784	0.4768	0.2562	0.1222	0.0847	0.0715	0.1562
k=6	0.3179	0.5088	0.3361	0.1710	0.0926	0.0543	0.1468
k=9	0.4970	0.5947	0.5449	0.3242	0.1047	0.0681	0.1728
k=12	0.6046	0.6600	0.6383	0.4213	0.1133	0.0701	0.1834
k=15	0.8213	0.7006	0.8666	0.6071	0.1557	0.0585	0.2142
k=18	0.8600	0.7348	0.8761	0.6437	0.1390	0.0773	0.2162
k=21	0.8356	0.7648	0.8529	0.6525	0.1274	0.0557	0.1831
k=24	0.6621	0.7911	0.6832	0.5405	0.0889	0.0327	0.1216
Import cost shock							
	Total	Intensive margin			Extensive margin		
	$\Delta\widehat{p}_{t+h}$	$\overline{\lambda}_h$	$\Delta\widehat{p}^*_{t+h}$	Total	Observable	Selection	Total
	(1)	(2)	(3)	(2)×(3)	(4)	(5)	(4)+(5)
k=0	0.1523	0.2030	0.2834	0.0575	0.0867	0.0080	0.0948
k=1	0.1977	0.2842	0.3372	0.0958	0.0934	0.0085	0.1019
k=2	0.2181	0.3512	0.3517	0.1235	0.0893	0.0053	0.0946
k=3	0.2512	0.3968	0.4021	0.1596	0.0987	−0.0070	0.0916
k=4	0.2606	0.4381	0.4060	0.1779	0.0920	−0.0092	0.0827
k=5	0.2501	0.4768	0.3885	0.1853	0.0787	−0.0138	0.0649
k=6	0.2407	0.5088	0.3819	0.1943	0.0695	−0.0232	0.0463
k=9	0.2611	0.5947	0.3719	0.2213	0.0580	−0.0182	0.0398
k=12	0.2529	0.6600	0.3465	0.2287	0.0512	−0.0270	0.0242
k=15	0.1812	0.7006	0.2226	0.1560	0.0347	−0.0095	0.0253
k=18	0.1909	0.7348	0.2153	0.1582	0.0337	−0.0010	0.0327
k=21	0.1986	0.7650	0.2098	0.1605	0.0323	0.0058	0.0381
k=24	0.1981	0.7911	0.2058	0.1628	0.0292	0.0061	0.0353

Note: Decomposition into intensive and extensive margin components according to 8, whereas the latter is further split into contributions from the observable shocks and from endogenous selection into price changes. We use the following estimated elements (1) Pass-through estimates using OLS incl. zeros; (2) measured frequency of cumulated unconditional price changes shown in Figure 1; (3) selection bias corrected estimates already presented in Table 3; (4) = difference between OLS estimates without zero price changes and (1) scaled by (2); (5) selection = OLS estimates on non-zero price changes minus selection bias corrected β^h from (3), scaled by (2).

fairly well by a time-dependent model of price stickiness. Nevertheless, the contribution of state-dependent adjustment, depicted by the dotted-dashed blue line, is non-trivial, and for both shocks is

Figure 8: Margin decomposition

(a) Energy cost shock (b) Import cost shock



Note: Estimated coefficients of good-level cumulative price changes on two shocks, both of which are constructed as lagged share of the input in total cost and the input price change. The black solid line is estimated using linear local projections on the full sample including zero price changes. The dashed lines split this average adjustment into an intensive and extensive margin according to columns (2)×(3) and (4)+(5) in Table 4. 95% confidence intervals for the state-dependent contribution are obtained by regressing the difference between all price changes and the fitted values of the selection corrected model on the shocks, where standard errors are clustered as usual.

actually the largest in the first few months after both shocks, when it is also marginally statistically significant (at the 10% level as shown by the dotted blue lines depicting 95% confidence bands).[27]

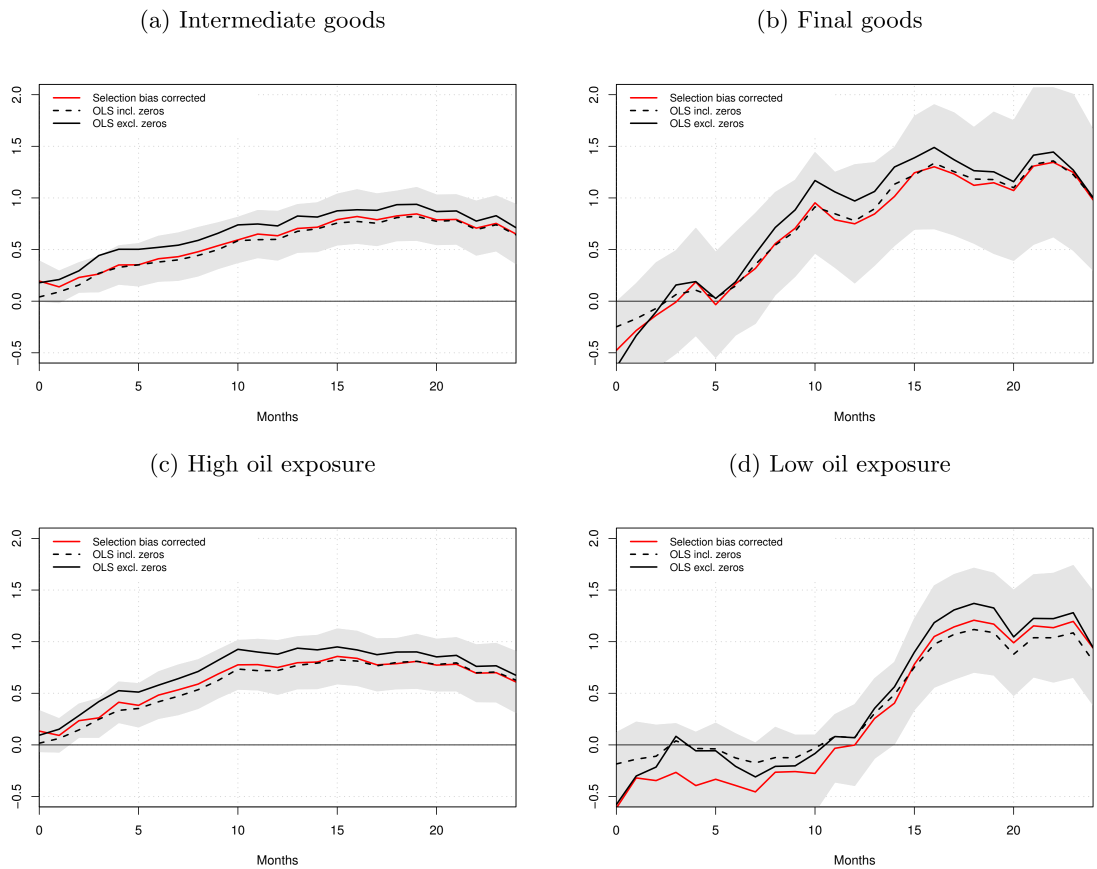
4.4 Pass-through heterogeneity

Next, we turn to understanding the heterogeneity in the response of reset prices to the two shocks. First, we investigate the reason why adjustment is delayed in response to the energy shock even for reset prices. It seems reasonable that the gradual adjustment may be due to the slow transmission of the oil-supply driven energy shock along the supply chain, with up-stream sectors and sectors more exposed to oil and energy (directly and indirectly) reacting faster than downstream sectors and less exposed sectors. Therefore, Figure 9 reports the results when we split prices into sectors as follows: the top row of the figure shows cumulated price responses of upstream (intermediate) and downstream (final) goods, on the left and right columns, respectively; the bottom row shows cumulated price responses of sectors with a different exposure to oil (and petroleum products), on the left and right columns, respectively.

As before, the figure shows OLS estimates for both non-zero price changes and also including unchanged prices (solid and dashed black lines, respectively), as well as selection-corrected elastici-

[27]Specifically, the state-dependent component and its confidence bands were obtained by regressing the difference between zero and non zero price changes and fitted values for both shocks from (7) on X_{ij}. These estimates of the state-dependent component are very similar to those in Table 4.

Figure 9: Energy cost pass-through by product class and oil exposure

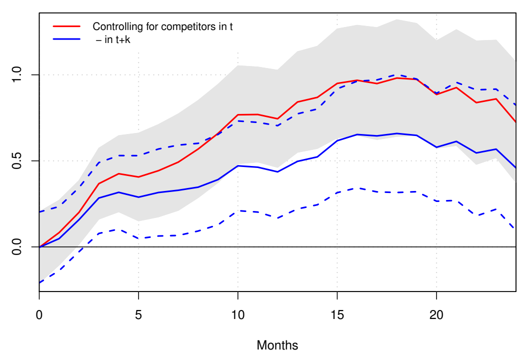


Note: Estimated coefficients of good-level cumulative price changes on two shocks, both of which are constructed as lagged share of the input in total cost and the input price change. We additionally interact the shocks with an indicator, estimating a β^h for each value of the indicator. For the top row, we interact with a category that describes whether the good is classified as a final or intermediate good according to Table 5 (whereas capital and energy goods are excluded). clarifies the details. In the bottom row, we group firms according to how "close" their 2-digit (in some cases 3-digit) NACE production sector is to movements in the oil price. To that end, we take the lagged input-output tables of the Danish economy and compute, for each sector, the share of oil sector in total inputs, taking into account the indirect exposure to oil from other intermediate inputs. We then split sectors at the median of the indirect exposure measure. 95% confidence bands in grey; standard errors are clustered at the firm level and corrected for first-step uncertainty.

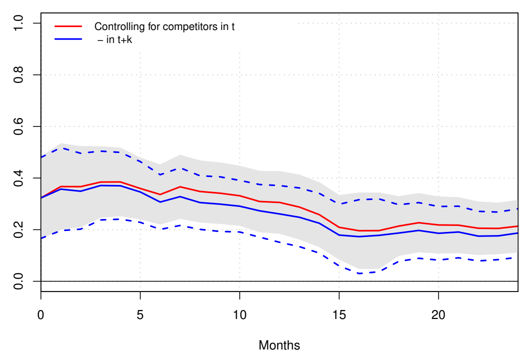
ties (red solid line with 95% confidence bands in grey). Figure 9 shows that prices of products with a lower oil intensity, and to some of final goods too, respond much later than those of products with higher oil intensity and intermediates. Focusing on biased-corrected estimates (the red solid line), the response of prices of final goods is positive and statistically significant only in the second half of the year after the shock, while the price response of low oil intensity sectors becomes significantly positive well into the second year after the shock. Moreover, price responses are very similar across all sectors at the end of the second year after the shock, in line with its pervasive propagation to

Figure 10: Pass-through estimates controlling for competitor price changes between t and $t + h$

(a) Oil price shock



(b) Import cost shock



Note: Estimated coefficients of good-level cumulative price changes excluding zero price changes on two shocks, both of which are constructed as lagged share of the input in total cost and the input price change. While the based controls for competitor price changes only in time t ($\overline{p}_{i,-j,t}$), the blue line shows estimates when controlling for $\overline{p}_{i,-j,t+h} - \overline{p}_{i,-j,t-1}$ at each horizon. 95% confidence bands in grey; standard errors are clustered at the firm level and corrected for first-step uncertainty.

total variable costs over time shown in Figure 3b above. Nevertheless, the left column of Figure 9 shows that the response of prices of intermediates and products with a higher energy intensity still builds up over time, peaking only after 12 months at values that are significantly larger than those in the first few months.

While the delayed pass-through in response to energy shocks seemingly reflects their slow propagation across different sectors, the question still remains of whether pass-through in the medium run is really different between the two shocks, as seemingly implied by the different levels of elasticities at the longer horizons, or it rather reflects different properties of the shocks, such as the more widespread impact of the energy shock across many firms. In order to shed light on this question, we now include among regressors the changes in competitors' prices between t and $t + h$, $\overline{p}_{i,-j,t+h} - \overline{p}_{i,-j,t-1}$. Specifically, Figure 10 compares OLS estimates of pass-through to both shocks in the case of non-zero price changes when controlling for cumulated competitor price changes at each horizon h (blue solid line with dashed lines indicating 95% confidence bands) with those in the baseline case shown above (red solid line with gray area indicating 95% confidence bands).

While the right-hand side panel shows that responses to import shocks in the two specifications are very similar, the left-hand side panel shows that controlling for $\overline{p}_{i,-j,t+h} - \overline{p}_{i,-j,t-1}$ substantially lowers responses to energy shocks. Moreover, we cannot reject the hypothesis of equality of the pass-through coefficients of the two shocks for the longer horizons (i.e. after $h = 22$). This suggests that differences in pass-through across shocks are mainly due to the differing dynamics in the responses of competitors prices to the two shocks, but that eventually pass-through converges to levels that are not too heterogenous across the two shocks.

We conclude our analysis by turning to the analysis of a second source of heterogeneity, resulting either from firm size as proxied by the number of employees or by the number of products (but found little evidence of heterogeneity in price adjustment along this dimension).[28] Previous contributions (e.g. Amiti et al. (2019)) have found that larger firms tend to react less to costs shocks to protect their market shares, consistent with significant market power. Figures 11 report estimates of price adjustment for the non-zero price changes of firms with less than 100 employees (first column), and with more than 100 employees (second column), the same cut-off used by Amiti et al. (2019); the top row shows the results for the energy cost shock, while the bottom row shows results for the import cost shock (again a solid black line denotes OLS estimates and a solid red line denotes bias-corrected estimates).

Smaller firms react by more than larger firms, as responses in the first column are consistently above those in the second column. In response to the energy cost shock, the red (and also the black) solid line in top left graph reaches well above 0.5 after 3-4 months and quickly converge to around 1, and is always statistically significant, while the red solid line in the top right graph is statistically insignificant for several months after the shock, and always well below 1. In the bottom row, in response to the import cost shock, all estimates on the left-hand side are mostly contained in the range 0.4-0.6, while on the right-hand side the black and red lines converge to 0.2 from $h = 15$ on. Nevertheless, differences in the responses of large and small firms after 12 months are statistically significant only in the case of import cost shocks. This suggests that the fact that their competitors are barely reacting to the latter shocks, induces larger firms to pass-through a smaller fraction of the cost change.

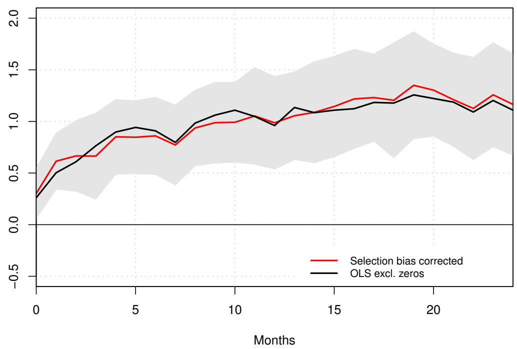
4.5 Robustness

We conclude this section on our empirical results by presenting a range of robustness checks, all of which can be found in Appendix C. First, it could be that by using the firm-level import and energy shares we are introducing measurement error in marginal costs at the good level; this could result in downward bias in our estimates. Nevertheless, results do not change when we re-run our estimates aggregating all good price changes at the firm level, arguably reducing measurement error. As shown in the Figure 18), we still find pretty much the same cost pass-through for firm-level prices as for good-level prices in Figure 7.

[28]Differences in the number of goods could be important in driving heterogeneity in price adjustment according to the theories we discussed above emphasizing complementarities in fixed costs in re-optimizing prices.

Figure 11: Oil price pass-through by firm size

(a) < 100 employees



(b) ≥ 100 employees

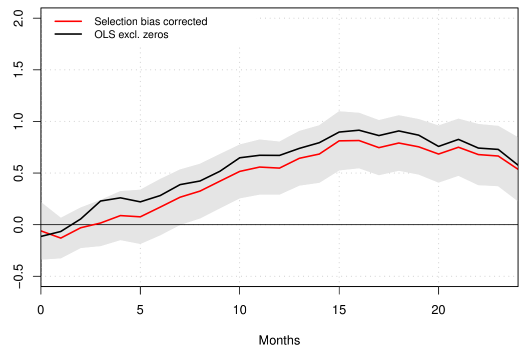
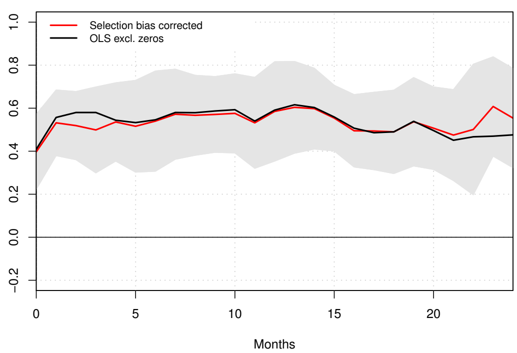
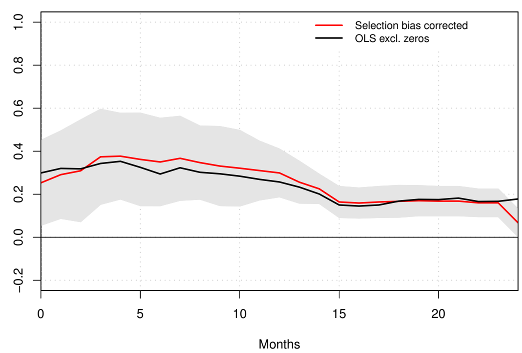


Figure 12: Import cost shock by firm size

(a) < 100 employees



(b) ≥ 100 employees



Note: Estimated coefficients of good-level cumulative price changes on two shocks, both of which are constructed as lagged share of the input in total cost and the input price change. We additionally interact the shocks with a dummy for whether the firm has more or less than 100 employees at the time of the shock. 95% confidence bands in grey; standard errors are clustered at the firm level and corrected for first-step uncertainty.

Second, if the cost shock leads to the termination of products (for which pass-through would potentially have been higher) or even firm exit, we might overestimate the structural pass-through. Figure 19 presents the estimates of all three models only on the subsample of goods that we observe for all horizons between $h = 0$ and $h = 24$.

Third, we estimate separate coefficients for domestically sold and exported goods. Both impulse responses are depicted in Figure 20. The response of export prices to the energy shock is statistically significantly different from that of domestic prices; nevertheless, our baseline results mostly reflect the response of the latter.

We show, fourth, the difference between positive and negative cost shocks in Figure 21. The results suggest that prices adjust more to cost push shocks than to cost decreases. The difference is not statistically significant and we cannot exclude the fact that the heterogeneity reflect differences in the (persistence of the) shock itself, rather than structural differences in pass-through.

Fifth, as in the first step we estimate a multinomial logit model, we are relying on the assumption that residuals across the decision to decrease, increase or keep a price constant are uncorrelated. Nevertheless, in Table 8 we show that when we relax this assumption and estimate instead a multinomial probit model in the first step, using in the second step the selection correction formulas in Glewwe (1993),[29] Our results concerning the biased-corrected estimates of pass-through are broadly unchanged. Specifically, while we statistically reject that correction terms do not belong in the specification, still estimated selection contributes only little to price responses conditional on adjustment.

Finally, as a further check on our selection correction procedure, we also re-estimated our two-step model for energy shocks only by dropping all firm-level cost measures (namely total variable costs, import costs, labor costs and even sales). The fact that these cost measures are now included among unobserved shocks could result in a larger bias in our estimates, thus providing a check of the robustness of our correction procedure (since they have a fair amount of explanatory power for price changes as shown in the R2 reported in the last column of Table 3). Nevertheless, Figure 22 in the appendix shows again that our baseline results for energy shocks remain broadly unchanged desping omitting all observed firm-level costs, both in the extensive and intensive margin.

5 Conclusions

This paper studies price adjustment in a novel monthly dataset of prices of multiproduct firms, merged with firm-level balance sheet and cost data. The theoretical literature on price setting has pointed out that the interdependence between the decision to whether or not change prices (the degree of state dependence) and the actual amount by which prices change (the intensive margin) is key to understand inflation dynamics and contributes to determine the real effects of monetary policy. Specifically, in standard menu costs models, firms change those prices that are most misaligned and furthest from their optimal values, resulting in a so-called selection bias that attenuates monetary non-neutrality.

We exploit the richness of our dataset to estimate the pass-through of shocks to firm-level import and energy costs (due to oil supply shocks) along extensive and extensive margins, modelling

[29] See Appendix E

them jointly to address endogenous selection bias due to state-dependent pricing decisions. In our first step, we model the probability of price changes over horizons from 1 to 24 months (state dependence), by using a flexible multinomial logit model. We find that there is evidence of synchronization of adjustment decisions within firms, especially as the number of goods increases, consistent with models of multiproduct firms. We also find evidence of state dependence as the probability of price adjustment over time is affected by our cost shocks, but also by aggregate inflation and even exchange rates.

Using first-stage estimates to correct for selection bias, we find that state dependence however does not translate into a large bias in the intensive margin conditional on price adjustment, nor in a large contribution to average price adjustment by the extensive margin. Moreover, pass-through of energy and import cost shocks is quite heterogeneous across sectors, and firms of different size. Gradual adjustment to energy costs mainly reflects faster price responses in upstream, intermediate sectors and in sectors highly exposed to oil shocks both directly and indirectly. Our results thus provide micro-based evidence on the debate about the propagation of idiosyncratic and common shocks to aggregate inflation, since firm-specific import cost shocks elicit a faster adjustment than energy cost shocks, whose effects instead build up through the supply chain in line with the pipeline pressure view.

Finally, for import-cost shocks, pass-through of larger firms is lower than that of smaller firms, consistent with the presence of strategic complementarities in price setting.

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A Data appendix

A.1 Producer price micro data

A.1.1 Sampling

We use the confidential microdata underlying the Danish producer and import price index for commodities compiled by Statistics Denmark. The raw data covers the time period from January 1993 until June 2017. The producer and import price index for commodities is based on approximately 6'400 prices at the firm-good level per month across 1,050 different commodities, reported by selected producers and importers in Denmark, see also Statistics Denmark (2019). Approximately 3'500 prices are used for calculating the producer price index, approximately 2'900 prices are used for calculating the import price index. The most important firms within selected areas are requested to report prices in order to ensure that the producer and import price index covers at least 70 percent of Danish production and imports.

The population covers all commodities that are imported or produced in Denmark for the domestic market or export, with the exception of some well-defined exemptions. Some commodities are not included because the turnover is too small and some commodities are not included because of the nature of the commodities.

Statistics Denmark undertakes great efforts to adjust for quality changes and product substitutions so that only true price changes are measured. When a product is substituted, Statistics Denmark re-computes the base price, and therefore we are able to identify replacements. They constitute only 0.7 per cent of all prices changes (including zero price changes) and 0.8 per cent of all non-zero price changes. We include these in the baseline results we report, but control for identified product replacements in regressions. Goods are defined relatively narrowly in our dataset, as products are classified using the 8-digit combined nomenclature (CN). The first 6 digits of the CN codes correspond to the World Harmonized System (HS). We address breaks in product classifications by identifying changes in product codes within a firm which do not lead to a change in the price. The vast majority of identified breaks coincides with the months where Statistics Denmark re-defines product categories. The breaks constitutes only 0.04 per cent of all price changes (including zero changes), and *per construction* 0 per cent of all non-zero price changes. Similar to product replacements, we include these incidents in the baseline results we report, but control for identified breaks in regressions.

The prices used for the index are actual prices, which means that the prices must include all possible discounts. Therefore, list prices do not apply unless the prices never include discounts. A distinction is made between the prices of imported commodities and the prices of commodities for

Table 5: Price change statistics by broad economic category (BEC)

	# unique products	Frequency Mean	Frequency Median	Size Mean	Size Median
All	5'354	20.6	8.0	7.0	5.0
Consumer goods					
Food	1'081	32.4	15.3	6.9	5.3
Nondurable non-food	462	14.2	6.0	8.4	5.0
Durables	534	11.7	5.3	6.6	4.7
Intermediate goods	1'710	22.1	9.1	7.1	5.1
Energy	85	80.9	96.8	8.8	8.5
Capital goods	1'413	11.9	6.1	6.6	4.5

Note: Summary statistics by broad product categories, 2008-2017. We compute the mean at the product level first, based on which the mean/median is taken across products in the category, classified from HS codes using UN correspondence tables. Frequencies and size of price adjustments are in %.

the domestic market or the export market:

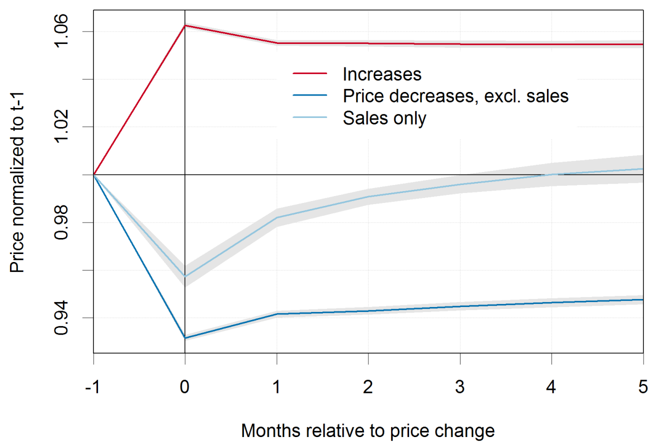
- Imported commodities: Actual transaction prices (in some cases transfer prices) c.i.f. excluding all duties and taxes on the goods as far as possible on the 15th day of the month. For the firms reporting import prices, we calculate a firm-level import price index using the equally weighted average log differences in each month.

- Danish commodities for the domestic market or export: Actual transaction price (in a few cases transfer prices) ex producer excluding VAT and excise duties as far as possible on the 15th of the month.

One advantage of this data is the relatively long time spans during which we observe uninterrupted price spells, allowing us to study dynamic pass-through at the good level. On average, the price of a good is reported for 115 subsequent months. During the time range we use in our pass-through analysis (2008m1-2017m6), a total of 5,354 product spells (at the firm-good level) can be identified, 79% of which we observe for at least 2 subsequent years. 30% of good id's can even be tracked along the entire sample of 9.5 years. Re-classification of products in January of 2009 (2014) leads to spikes in the exit and entry rate of products of 30% (9%), which we do not link because we do not observe quantities and are therefore unable to compute counterfactually weighted prices. In other months, half of entry and exit of products is driven by firm re-sampling, whereas smaller firms are re-sampled more frequently.

Products reported cover a broad set of goods representative of the Danish economy. The manufacturing sector makes up more than 75% of firms in the data and even more in terms of goods.

Figure 13: Typical price change paths



Note: Conditional on price changes in period t, this figure shows the average path of the price during the following subsequent 5 months. Sales are defined as price changes that are fully reverted after 1-3 months. Details are available in the data appendix.

The second largest industry is wholesale trading. Within manufacturing, machinery, food products, fabricated metal, plastic and computer and electronics are the most commonly found industries. We define sub-markets in terms of products sold at the 2-digit level of HS codes, which results in 74 product categories such as meat, pharmaceutical products, or furniture. Further, we link product identifiers to broad economic categories (BEC) according to UN correspondence tables and report price statistics of frequency and size of price adjustment for each category in Table 5.

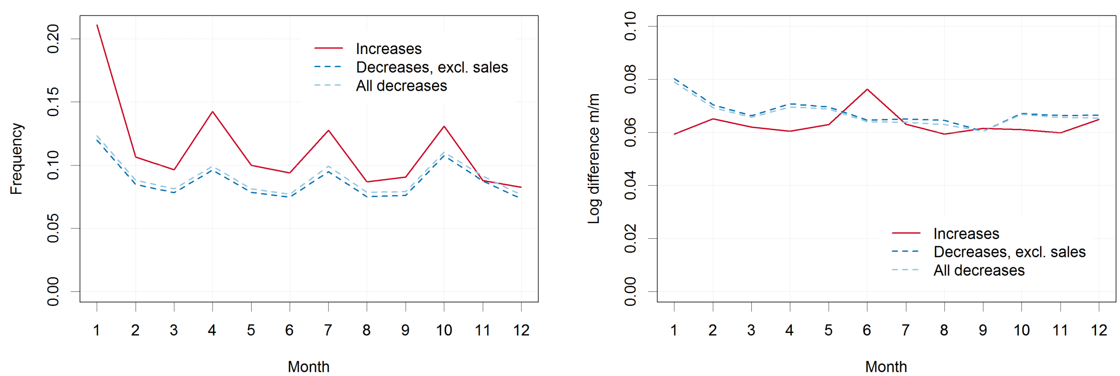
A.1.2 Further descriptive statistics

This Subsection contains a variety of descriptive statistics regarding price changes used, in one way or another, throughout the paper.

Temporary sales Typical price adjustments are rather persistent and close to permanent, as is shown in Figure 13 for both price increases and decreases. The latter are shown including and excluding sales as identified by "filter B" in Nakamura and Steinsson (2008). Not only is the typical "sales" price decrease less persistent by construction than the typical non-sales decrease, but it is also smaller. Because of this, and since we also identify less than 3.5% of all price drops as sales, we do not exclude sales from our econometric analysis.

Figure 14: Seasonality of frequency and size of price changes

(a) Frequency of price changes by month (b) Absolute size of price changes by month

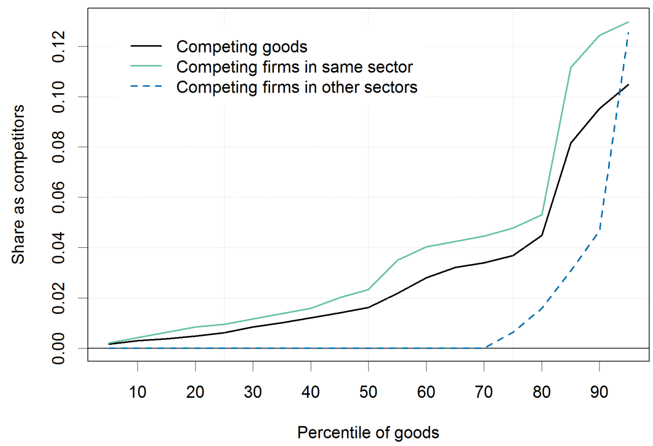


Note: Mean frequency of price changes of firms per month of the year. Price changes (particularly increases) are most frequent in January, with local peaks at the first month of any quarter. Sales remain quantitatively minor and do not have a sesonal pattern different from regular price decreases.

Seasonality We find a substantial seasonal component of PPI price changes, in striking similarity to Nakamura and Steinsson (2008). Figure 14 presents the median frequency (panel (a)) and the mean absolute size (panel (b)) of both price increases and decreases by month — whereas results for decreases are very similar whether we include or exclude sales. Four results stand out. First, the frequency of price changes declines monotonically over the first three quarters, and then is roughly constant. Second, in all four quarters, the frequency of price changes is largest in the first month of the quarter and declines monotonically within the quarter with the exception of September. This gives rise to the pattern of local peaks in the frequency of price changes in January, April, July, and October. Third, price increases play a disproportionate role in generating seasonality in price changes. Producer prices are twice as likely to change and increase in January than on average in other months of the year. Fourth, seasonality is much less apparent in the mean size of price increases and decreases, and if anything follows a different pattern than in the price change frequency. Mean price increases are not larger in the months at the beginning of quarters, when the frequency is higher; price decreases are larger and more frequent in January.

Overall, these results suggest some time dependence of price changes, with possibly significant implications for the transmission of shocks. Olivei and Tenreyro (2007) show that the real effects of monetary policy in the US differ depending on the quarter of the year in which the shock hits. They argue that seasonality in the flexibility of wages can explain their empirical findings. Our result that a disproportionate number of price increases are recorded in January could point to similar effects in Denmark and even in the euro area, as Álvarez et al. (2006) also find that prices are

Figure 15: Competition intensity across goods



Note: A competing good is a good sold in the same (of 74) product sectors defined by the first two digits of the HS code. We count the number of other goods and competing firms in the same product sector for every good, and divide it by the total amount of goods and firms in the sample in the respective period.

significantly more likely to change in January in the euro area. However, the size of price changes does not seem to be much larger in January, pointing to other mechanisms beyond large seasonal changes in firms' costs or demand.

Competition As we have laid out in the paper, firms' pricing decisions are a function of their competitors prices under imperfect competition. The 942 firms we include in the analysis compete on different markets. We define competitors to be firms that sell products in the same 2-digit category of the Harmonized System in the same month. 74 such product sectors are identified. The average number of competing firms in each sector is 42, whereas the first/second/third quantile of numbers of competitors for which we observe prices is 11/26.5/47. We will refer to the geometric average of all known firms in the same product sector as the price change of competitors.

Figure 15 illustrates the heterogeneity in the degree of competition across goods. We do observe competitor's prices even in the markets in which there is the least competition. On the other end, 20% of goods are sold in markets where they compete against up to 10% of all goods in the data. Furthermore, the dashed line underscores the network structure of the producer price data: Because firms operate in more than one product sector, 30% of products not only face direct competition from other firms in the same sector, but also indirectly from firms operating in the same and other markets. Our data allows us to analyze the strategic complementarities at play when cost shocks propagate along supply chains.

A.2 Firm registers

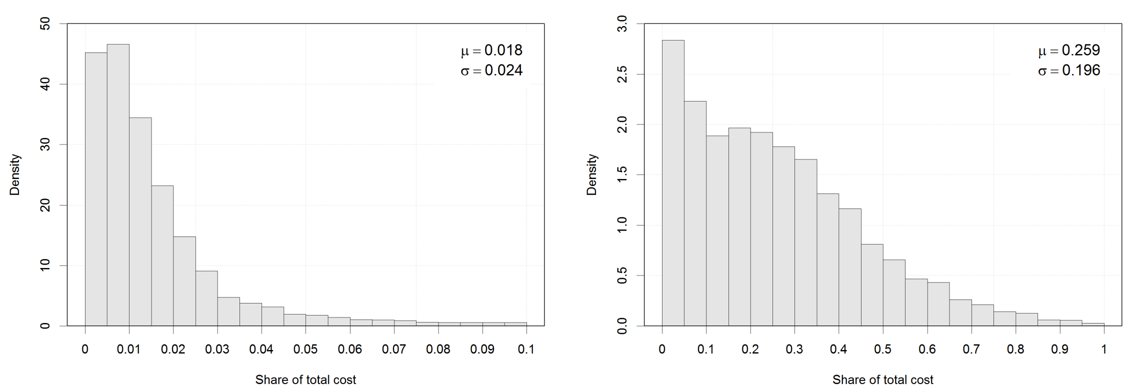
We combine the pricing data with annual firm-level data from Statistics Denmark's accounts statistics for the Danish business sector in the period from 1996 to 2016 (FIRE registers). A firm is identified at the enterprise level, i.e. the legal unit, see also Statistics Denmark (2017). The primary industries, the financial sector and the public sector are excluded.

The share of firm identifiers in the price data we match to accounting statistics lies between 89% (in 2008) and 99% (in 2017).

Income statement items we use include total sales and profits, from which we impute total cost. Firms report the total amount spent on purchasing energy throughout a year. The mean (median) spending on energy as a share of total cost is 1.7% (1.09%). Furthermore, we observe the number of employees in full-time equivalents, firm age (for a subsample of 81% of the firms), as well as expenditure on imported goods. We calculate the latter as a share of approximated total cost: The mean (median) import intensity is 27% (23.1%). The import intensity is available directly from the VAT registers at the monthly frequency.

Figure 16: Histograms of cost shares

(a) Energy share (b) Import share

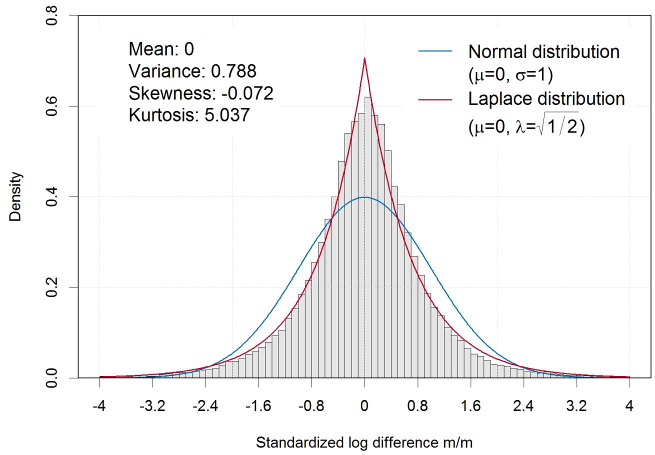


Note: Energy (from annual accounting statistics) and cost of imported inputs (from VAT declarations) as a share of total firm-level cost.

A.2.1 Monthly sales, purchases, and payrolls

For all firms covered by the Danish VAT system, we have information on purchases and sales, see also Statistics Denmark (2018). The data (referred to by Statistics Denmark by the mnemonic "FIKS") contains information on total sales and total purchases from 2001 to 2017, with the category of imported purchases reported separately starting in 2002.

Figure 17: Histogram of standardized cost changes



Note: Histogram of changes in variable cost measured as the sum of total intermediate purchases (domestic and import) and labor cost at the monthly frequency. We exclude zero-cost changes and cost changes smaller than 0.01% and superimpose a normal and Laplace distribution with unit variance.

The monthly frequency of this dataset allows us to leverage the high frequency of the pricing data. However, some firms do not report on a monthly basis, whereas the annual turnover of a firm determines its VAT declaration frequency. The frequency is monthly if the amount exceeds DKK 50 million, quarterly in the interval between DKK 5 million and DKK 50 million, and half-yearly if it is less than DKK 5 million (and above DKK 50,000). Quarterly and semi-annual data are recalculated and spread onto months by Statistics Denmark using information from firms with monthly VAT reporting in the same industry (at the DB-127 level). Due to the universal nature of the VAT registers, we match more than 99% of good-month observations for the time range used in this paper (2008m1-2017m6).

Furthermore, we use monthly payrolls from the BFL registers starting in January 2008. Danish firms register hours worked by and total compensation of employees in the tax authority's *e-Indkomst* with the payment of every remuneration. While the raw registers are matched employer-employee data, we aggregate monthly wage payments and hours to the level of the firm id and link changes to the ppi data. The share of firm identifiers in the price data we match to accounting statistics lies between 89% (in 2008) and 99% (in 2017).

Figure 17 shows the distribution of monthly changes of these material and labor expenditure cost.

B Constructing marginal cost shocks

We construct two cost shocks as shift-share combinations. Oil price shocks move the price of energy for production in Denmark and, interacted with the (lagged) firm energy cost share, constitute changes in marginal cost of the producers. We proceed similarly for the import cost shock, where both the price and cost share are measured at the firm level. Since the distribution of cost shares for energy and imported inputs is shown in Figure 16, this Section's focus is on the validation of the Baumeister and Hamilton (2019) series of oil supply shocks as a driver of the aggregate Danish energy price. Furthermore, we present descriptive statistics on the (joint) distribution of the two shocks.

B.1 Energy cost shocks

The common shock we consider in this paper is a shock to the price of energy. Changes in the price of energy arguably have a strong demand component, with different implications for the behavior of firms' prices. We address this issue in two ways: First, we consider oil price changes as a predictor of energy price changes. Since Denmark is a small open economy, changes in domestic demand are unlikely to systematically affect the price of Brent crude oil. Still, domestic and world demand for oil might be correlated, which is why we rely on a series of oil supply shocks provided by Baumeister and Hamilton (2019) instead. They estimate a VAR with oil prices, production and inventories as well as world industrial production, identified using prior information to distinguish between oil supply and consumption shocks. The prior conjectures that short-run elasticities of production are small. The prior mode is 0.1 (whereas the resulting posterior has a mode of 0.15). Impulse responses show that a one-standard deviation shock to oil production increases the oil price by 3%. When replicating this elasticity for the time period of our sample, we find it to be higher (4.86%). Table 6 reports the results of a projection of the end-of-month Brent crude oil price on the BH supply series. Baumeister and Hamilton (2019) find that the lion's share of oil price movements is indeed driven by supply shocks, and that inventories play a minor role in the transmission of this shock, which further motivates our approach.

Note that the pass-through to the world oil price is instantaneous, and the coefficients of regressions of current oil price changes on lagged realizations of the shock are insignificant.

The cost measure for which we want to estimate the pass-through to producer prices is the price of domestic energy, which apart from oil and petroleum products includes electricity and heating. The index is constructed by the Danish statistical office using a subsample of our PPI data. Its correlation with the oil price changes is 0.46. As the right-hand side columns of table 6 shows, the

Table 6: Elasticities of oil and energy prices with respect to oil supply shocks

	Δp^O		Δp^E	
BH oil supply shock	-4.86^{***}	-4.87^{***}	-1.57^{***}	-1.52^{***}
— $_{t-1}$		-0.55		-1.40^{***}
— $_{t-2}$		-0.63		-0.25
— $_{t-3}$		0.39		-0.15
— $_{t-4}$		0.82		0.17
— $_{t-5}$		-0.09		-0.25
— $_{t-6}$		0.15		-0.11
N	114	114	114	114
R2	0.37	0.39	0.16	0.32

Significance levels: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Note: Dependent variables: Monthly log differences of the price of Brent crude oil (Δp^O) and the Danish energy price index (incl. oil, electricity and heating) provided by the Danish statistical office (Δp^E). Regressions on contemporaneous and lagged values of the Baumeister and Hamilton (2019) oil supply shock series. Sample: 2008m1-2017m6.

domestic energy price reacts about a third of how oil prices do on impact, but the loading of the first lag of the BH oil supply shock is positive, indicating that it takes (a relatively short amount of time) for oil shocks to transmit to firm's energy cost.

We build the aggregate series by rescaling oil shocks based on the regression $\Delta \widehat{p}_t^E = \widehat{\beta}_0 + \widehat{\beta}_1 BH_t$ (i.e. column 3), normalized to have the variance of the original series Δp_t^E. This way, we can interpret the size of the shock as an exogenous shock to world supply of oil equivalent 1% increase in domestic energy prices.

B.2 Descriptive statistics of firm-level cost shocks

It results a panel of cost shocks that are small on average. Their distribution has a standard deviation of 0.28% of marginal cost but a very large kurtosis. Table 7 presents further moments of the distribution in the first column. The second and third column contain the import cost shocks and, for reference, the interaction of the labor cost share and the change in the average hourly wage paid in the firm according to the payroll.

The implications relevant for the pass-through analysis conducted in this paper are the following: Most realizations of both cost shocks are small and have a large mass around zero. For the energy cost shock, the 10% tails imply a marginal cost shock of +/- 0.18%. For the import cost shock, many realizations are zero because the firm reports unchanged prices in the import wave of the PPI survey, but 10% of the realizations are -1.1% or less.

The second feature is that the cross-sectional variance of import cost shock is considerably larger. Because all firms experience the same change in the energy price index, all variation stems from the distribution of energy intensities. If we quantify this commonality by means of a principal component analysis, we find that the first principal component explains 92% of the variance of all shocks. The import cost shock is much more idiosyncratic. This first principal component only accounts for 10% of the variation.

The third important feature of the shocks we construct is depicted in the bottom panel of Table 7. The correlation between the energy and import cost shocks, which we include jointly in our regressions, is virtually zero.

Table 7: Input cost changes

	$\phi^E_{jt-1}\Delta\widehat{p}^E_t$	$\phi^M_{jt-1}\Delta p^M_{jt}$	$\phi^L_{jt}\Delta w_{jt}$
Mean	0.0002	0.0071	0.0639
Standard deviation	0.2828	1.1135	3.7578
Kurtosis	51.607	4.778	69.8568
q1	-0.9021	-2.4861	-10.707
q10	-0.1896	-1.1071	-2.6272
q25	-0.0519	0.0000	-0.8268
q50	0.0053	0.0000	0.0216
q75	0.0626	0.0000	0.9197
q90	0.1803	0.0031	2.7872
q99	0.7860	2.6980	11.2326
Variance across firms*	0.0629	1.2050	11.3978
of which explained by 1^{st} PC	0.9193	0.1020	0.1647
— 2^{nd} PC	0.9586	0.1938	0.2833
— 3^{rd} PC	0.9746	0.2762	0.3783
Correlation with $\phi^E_{jt-1}\Delta\widehat{p}^E_t$		0.0057	-0.0047
— with $\phi^M_{jt-1}\Delta p^M_{jt}$			0.0028

Note: Selected moments of distribution of input cost shocks. Shares ϕ denote the lagged inputs expenditure as a share of total cost in the previous year, and Δp is the log difference of the respective input price. Moments are reported in % shocks to the firm's marginal cost for all inputs. *To calculate dispersions of shocks across firms, we first compute the variance of a shock across firms in a given month, and then average across time periods. We report what (cumulated) share of the variance in the shock variable can be explained by the first few principle components.

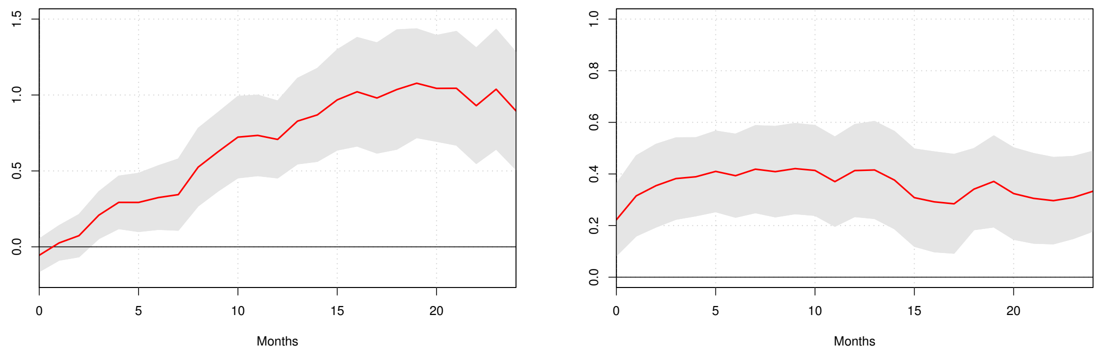
C Robustness

C.1 Firm-level pass-through regressions

Figure 18: Pass-through estimates at firm level

(a) Energy cost shock

(b) Import cost shock



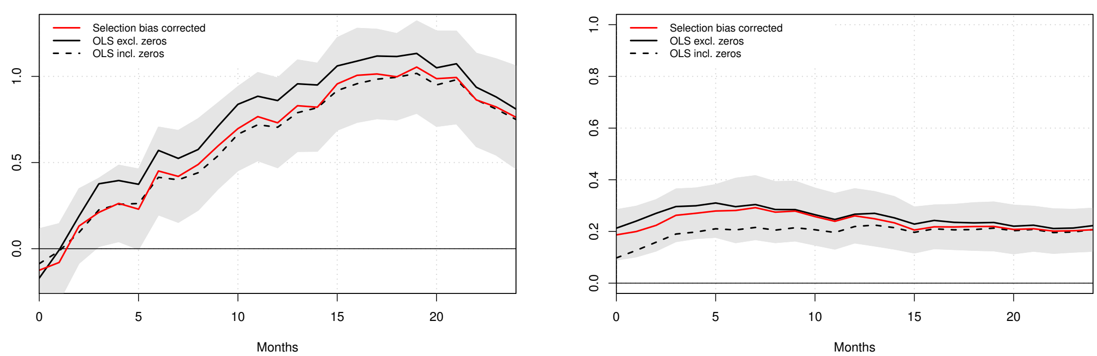
Note: Estimated coefficients of firm-level cumulative price changes on two shocks. We average period-by-period price changes $\Delta p_{jt} = 1/N \sum_i \Delta p_{it}$ of all goods within a firm, including those whose price do not change, and then cumulate price changes between $t-1$ and $t+h$, $\sum_{k=0}^{h} \Delta p_{j,t+k}$. We report 95% confidence bands based on standard errors clustered at the firm level.

C.2 Surviving products and firms

Figure 19: Pass-through estimates for "surviving" products only

(a) Energy cost shock

(b) Import cost shock

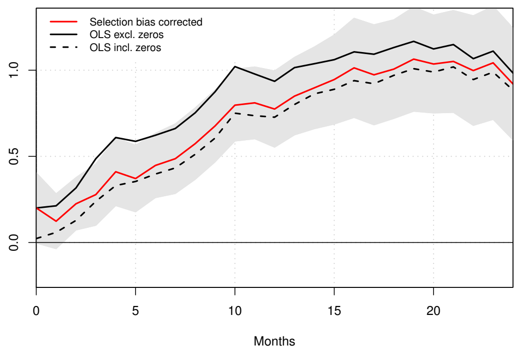


Note: Estimated coefficients of good-level cumulative price changes on two shocks, conditional on that the good price is still reported in $t+24$. We report 95% confidence bands based on standard errors clustered at the firm level.

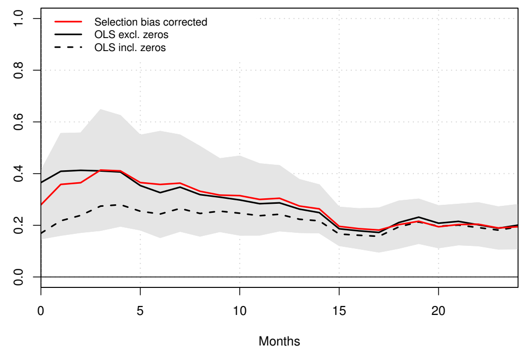
C.3 Domestic vs. export goods

Figure 20: Pass-through estimates by export status

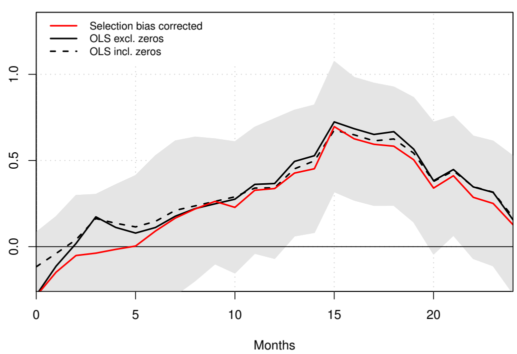
(a) Energy cost shock, domestic goods



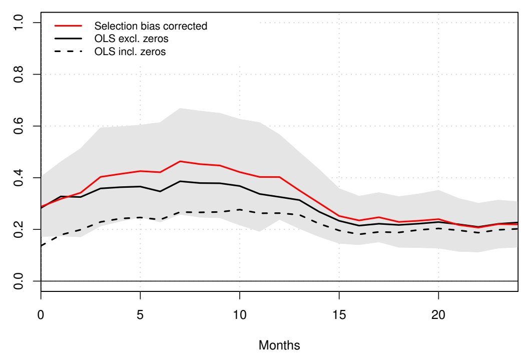
(b) Import cost shock, domestic goods



(c) Energy cost shock, export goods



(d) Import cost shock, export goods

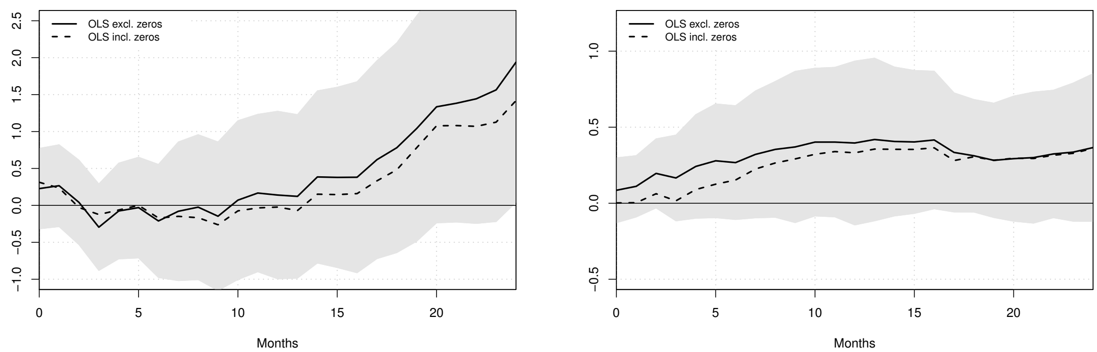


Note: Estimated coefficients of good-level cumulative price changes on two shocks, where the two shocks are interacted with a dummy for domestically sold goods. We report 95% confidence bands based on standard errors clustered at the firm level.

C.4 Positive vs. negative cost shocks

Figure 21: Difference of pass-through estimates between positive and negative shocks

(a) Energy cost shock (b) Import cost shock



Note: Estimated coefficients of good-level cumulative price changes on two shocks, where the two shocks are interacted with a dummy for positive cost shocks. We report 95% confidence bands based on standard errors clustered at the firm level.

C.5 Multinomial probit

Table 8: Estimated pass-through and selection coefficients

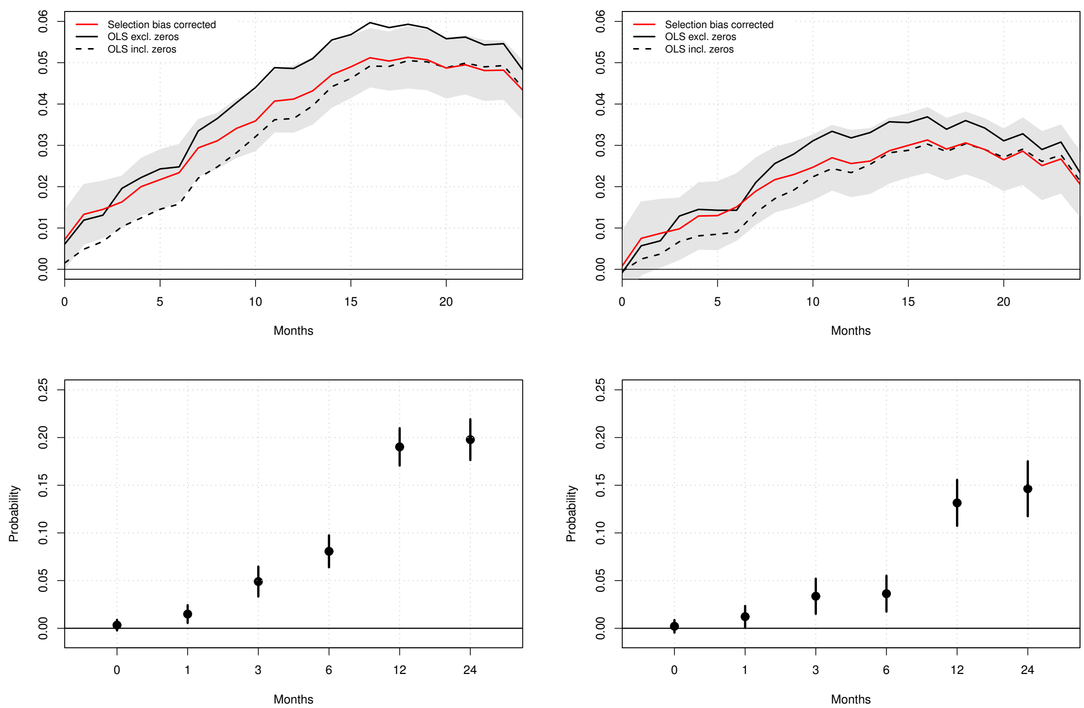
	$\phi^E_{jt-1}\Delta\widehat{p}^E_t$	$\phi^M_{jt-1}\Delta p^M_{jt}$	$\Delta\overline{p}_{i,-j,t}$	Neg. outcome $\frac{\sigma_{-1,0}}{\sqrt{\sigma^2_{-1,0}}}$	Neg. outcome $\frac{\sigma_{-1,0}-\sigma_{1,0}}{\sqrt{\sigma^2_{-1,1}}}$	Pos. outcome $\frac{\sigma_{1,0}}{\sqrt{\sigma^2_{1,0}}}$	Pos. outcome $-\frac{\sigma_{-1,0}-\sigma_{1,0}}{\sqrt{\sigma^2_{-1,1}}}$
k=0	0.0576	0.3403***	0.1925***	−0.0198***	−0.0606***	0.0123***	0.0600***
k=1	0.1046	0.3935***	0.2741***	−0.0211***	−0.0728***	0.0169***	0.0767***
k=2	0.1550*	0.3964***	0.2954***	−0.0168***	−0.0796***	0.0156***	0.0902***
k=3	0.2665**	0.4162***	0.3127***	−0.0163***	−0.0880***	0.0146***	0.0974***
k=4	0.4588***	0.4203***	0.2979***	−0.0158***	−0.0908***	0.0163*	0.1087***
k=5	0.4827***	0.3937***	0.2901***	−0.0017***	−0.0643***	0.0088***	0.0765***
k=6	0.4515***	0.3586***	0.2865***	0.0137***	−0.0781***	−0.0123**	0.0706***
k=9	0.6473***	0.3545***	0.2540***	−0.0023	−0.1020***	−0.0464***	0.0624***
k=12	0.7990***	0.3239***	0.2632***	−0.0129	−0.1194***	−0.0496***	0.0721***
k=15	1.0589***	0.2321***	0.2484***	0.0156	−0.1094***	−0.0251	0.0960***
k=18	1.2500***	0.2685***	0.2660***	0.0927***	−0.0815***	0.0026	0.1403***
k=21	1.2348***	0.2518***	0.1988***	0.1402***	−0.0837***	0.0147	0.1465***
k=24	1.0530***	0.2603***	0.2131***	0.2658***	−0.0791***	0.0427	0.1647***

Significance levels: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Note: Coefficients of the price response to own cost shocks ($\frac{1}{1+\Gamma}$ in Equation 3) and competitor prices in t as the control variable. Columns 4 through 6 contain the coefficient on the selection terms constructed as in Section E.

C.6 Excluding firm-level controls

Figure 22: Oil supply shock pass-through without firm-level controls

(a) 1993-2017 (b) 2008-2017



Note: Estimated coefficients of good-level cumulative price changes on the oil supply shock, omitting all control variables that are measured at the firm level, including firm-level cost shares. The lower panels depict, for selected horizons, the marginal effects of the shock on price increases. We report 95% confidence bands based on standard errors clustered at the firm level.

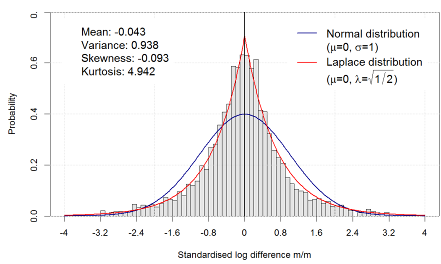
D Multiproduct firms (not for publication)

This Section relates our work to several aspects of the literature on multiproduct firms. First, Section D.1 shows the familiar, sector-normalized price change distributions when firms are grouped by the number of products they sell, similar to Alvarez et al. (2016) and Wulfsberg (2016). Second, we show statistics of frequency, sign and size of price changes in Figure 24, replicating Bhattarai and Schoenle (2014). Third, Table 9 bridges the gap between the multinomial logit of Bhattarai and Schoenle (2014) on price synchronization and our Table 2 introducing the marginal cost shocks.

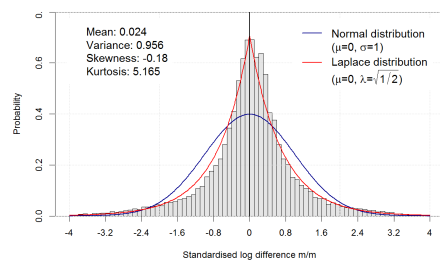
D.1 Moments of price change distributions by number of products

Figure 23: Histogram of price changes by number of products

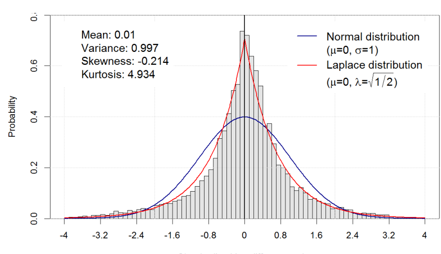
(a) Single-product firms



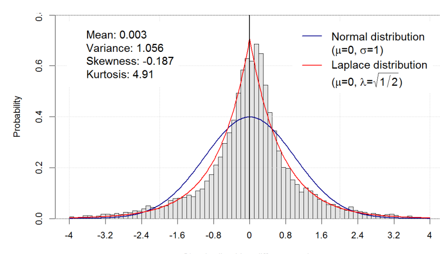
(b) 1-3 goods



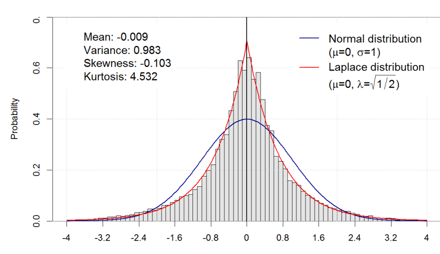
(c) 3-5 goods



(d) 5-7 goods



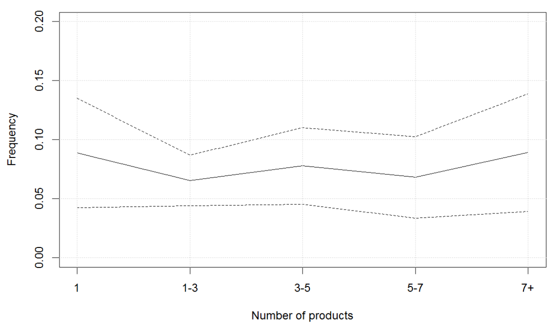
(e) More than 7 goods



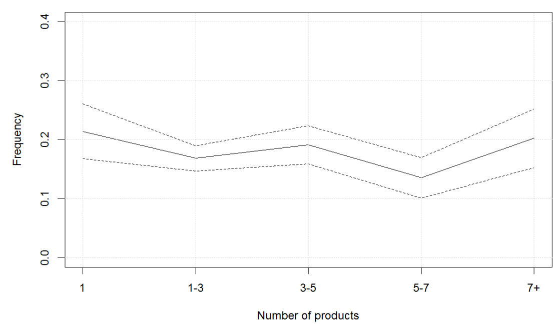
Note: We normalize price changes larger than 0.1% for each 2-digit HS code by subtracting the mean and dividing by the standard deviation within the sector. The plots show the histograms of these normalized price changes, as well as superimposed normal and Lapalce distributions with unit variance. Firms are binned according to the average number of products reported throughout the sample.

Figure 24: Price adjustments by number of products

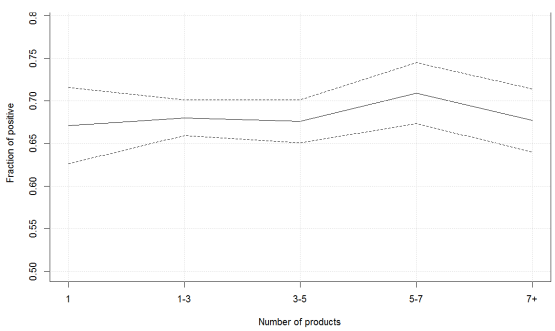
(a) Median frequency of price change



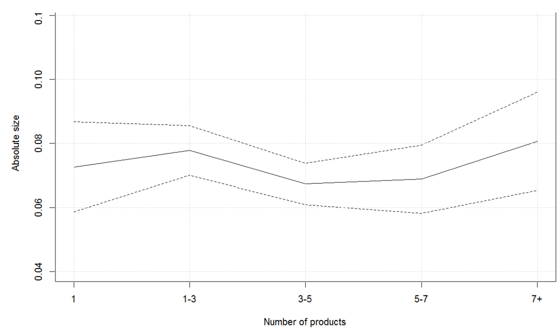
(b) Mean frequency of price change



(c) Mean fraction of positive price changes



(d) Mean absolute size of price change



Note: We first calculate mean frequencies and size of non-zero price changes at the good level, and aggregate over all products in firms of a particular bin. Firms are binned according to the average number of products reported throughout the sample

Table 9: Multinomial logit, price synchronization

	All	1-3	3-5	5-7	7+
Marg. effect on probability of increase					
Fraction of pos. price changes in firm	6.18***	4.37***	5.96***	6.31***	8.30***
	(0.00)	(0.03)	(0.00)	(0.05)	(0.05)
Fraction of neg. price changes in firm	2.79***	2.07***	2.59***	2.65***	2.81***
	(0.00)	(0.03)	(0.00)	(0.06)	(0.06)
Fraction of pos. price changes in industry	0.35***	0.46***	0.51***	0.214**	0.26***
	(0.03)	(0.07)	(0.07)	(0.08)	(0.06)
Fraction of neg. price changes in industry	0.044	0.053	−0.125*	−0.134	0.153*
	(0.02)	(0.08)	(0.06)	(0.07)	(0.06)
Avg. price change in firm, excl. good	0.10***	0.095*	0.04***	−0.015	0.38***
	(0.00)	(0.04)	(0.00)	(0.05)	(0.09)
Avg. abs. price change in firm, excl. good	−0.02***	0.005	0.05***	0.015	−0.251**
	(0.00)	(0.05)	(0.00)	(0.05)	(0.09)
Avg. price change in industry, excl. firm	0.27***	0.24***	0.154**	0.101	0.38***
	(0.03)	(0.05)	(0.05)	(0.06)	(0.06)
CPI, log difference	0.69***	0.695*	0.960**	0.261	0.427
	(0.16)	(0.28)	(0.31)	(0.32)	(0.33)
Marg. effect on probability of decrease					
Fraction of pos. price changes in firm	2.44***	1.85***	2.26***	2.12***	2.76***
	(0.00)	(0.03)	(0.00)	(0.05)	(0.06)
Fraction of neg. price changes in firm	3.95***	2.57***	3.87***	4.01***	5.26***
	(0.00)	(0.03)	(0.00)	(0.05)	(0.05)
Fraction of pos. price changes in industry	0.14***	0.038	−0.117	0.040	0.22***
	(0.03)	(0.07)	(0.06)	(0.07)	(0.05)
Fraction of neg. price changes in industry	0.41***	0.62***	0.58***	0.44***	0.26***
	(0.02)	(0.07)	(0.05)	(0.06)	(0.05)
Avg. price change in firm, excl. good	−0.09***	−0.066	−0.04***	−0.038	−0.261**
	(0.00)	(0.04)	(0.00)	(0.04)	(0.09)
Avg. abs. price change in firm, excl. good	0.02***	−0.004	0.04***	0.041	−0.079
	(0.00)	(0.04)	(0.00)	(0.04)	(0.08)
Avg. price change in industry, excl. firm	−0.25***	−0.22***	−0.098	−0.137**	−0.34***
	(0.03)	(0.05)	(0.06)	(0.07)	(0.07)
CPI, log difference	−0.460**	−0.536*	−0.627*	−0.612*	0.170
	(0.14)	(0.26)	(0.28)	(0.29)	(0.30)
N	599310	157652	151956	112730	161751
R2	0.404	0.445	0.437	0.473	0.369

Significance levels: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: Marginal effects (in percentage points) of a one standard deviation change in the regressor from the mean on the probability of increasing and decreasing the price relative to not changing the price. Exception: 1% in CPI inflation. Standard errors in parentheses. Further controls (not reported): Firm size, dummies for product replacement, sales, and exports, month fixed effects.

E Selection bias corrections using a multinomial probit model (not for publication)

Because it allows for arbitrary correlation between the error variables, we set up a version of our 2-step estimation approach where the first step uses a multinomial probit in place of the multinomial logit introduced in the main body of the paper.[30] We let Equation 5 be a latent variable model whose residuals $\eta_{ij,m,t+h}$ follow a multivariate normal distribution with covariance matrix Σ. Based on the cateogry chosen by the firm, the model identifies the model parameters relative to the reference category of no price change ($m = 0$), meaning $\beta_1 - \beta_0$ as well as $\beta_{-1} - \beta_0$. By the same token, the covariance matrix is reduced to a 2×2 matrix with the following elements:

$$\begin{bmatrix} \sigma^2_{1,0} & . \\ \sigma_{-1,0/1,0} & \sigma^2_{-1,0} \end{bmatrix}$$

Glewwe (1993) provides the analytical terms for bias correction in what we refer to as the second (linear) step of our pass-through estimation, which we apply to our setting and notation. Considering we want to correct for selection into decreasing and increasing prices from the reference of $m = 0$, we need to make the following adjustments depending on the actual option chosen

$$\begin{aligned} p_{ij,t+h} - p_{ij,t-1} = {} & \beta^h X_{ijt} \\ & + \frac{\sigma_{1,0}}{\sqrt{\sigma^2_{1,0}}} \times 1[m=1] \frac{\phi\left(\frac{\left[\gamma^h_1 - \gamma^h_0\right]' z_{ijt}}{\sqrt{\sigma^2_{1,0}}}\right)}{BN\left(\frac{\left[\gamma^h_1 - \gamma^h_0\right]' z_{ijt}}{\sqrt{\sigma^2_{1,0}}}, \frac{\left[\gamma^h_1 - \gamma^h_{-1}\right]' z_{ijt}}{\sqrt{\sigma^2_{-1,1}}}; \frac{\sigma^2_{1,0} - \sigma^2_{-1,0} + \sigma^2_{-1,1}}{2\sqrt{\sigma^2_{-1,1}\sigma^2_{1,0}}}\right)} \\ & \qquad \times \Phi\left(\frac{\frac{\left[\gamma^h_1 - \gamma^h_{-1}\right]' z_{ijt}}{\sqrt{\sigma^2_{-1,1}}} - \frac{\sigma^2_{1,0} - \sigma^2_{-1,0} + \sigma^2_{-1,1}}{2\sqrt{\sigma^2_{-1,1}\sigma^2_{1,0}}} \frac{\left[\gamma^h_1 - \gamma^h_0\right]' z_{ijt}}{\sqrt{\sigma^2_{1,0}}}}{\sqrt{1 - \frac{\left(\sigma^2_{1,0} - \sigma^2_{-1,0} + \sigma^2_{-1,1}\right)^2}{4\sigma^2_{-1,1}\sigma^2_{1,0}}}}\right) \\ & - \frac{\sigma_{-1,0} - \sigma_{1,0}}{\sqrt{\sigma^2_{-1,1}}} \times 1[m=1] \frac{\phi\left(\frac{\left[\gamma^h_1 - \gamma^h_{-1}\right]' Z_{ijt}}{\sqrt{\sigma^2_{-1,1}}}\right)}{BN\left(\frac{\left[\gamma^h_1 - \gamma^h_0\right]' Z_{ijt}}{\sqrt{\sigma^2_{1,0}}}, \frac{\left[\gamma^h_1 - \gamma^h_{-1}\right]' Z_{ijt}}{\sqrt{\sigma^2_{-1,1}}}; \frac{\sigma^2_{1,0} - \sigma^2_{-1,0} + \sigma^2_{-1,1}}{2\sqrt{\sigma^2_{-1,1}\sigma^2_{1,0}}}\right)} \\ & \qquad \times \Phi\left(\frac{\frac{\left[\gamma^h_1 - \gamma^h_0\right]' z_{ijt}}{\sqrt{\sigma^2_{1,0}}} - \frac{\sigma^2_{1,0} - \sigma^2_{-1,0} + \sigma^2_{-1,1}}{2\sqrt{\sigma^2_{-1,1}\sigma^2_{1,0}}} \frac{\left[\gamma^h_1 - \gamma^h_{-1}\right]' z_{ijt}}{\sqrt{\sigma^2_{-1,1}}}}{\sqrt{1 - \frac{\left(\sigma^2_{1,0} - \sigma^2_{-1,0} + \sigma^2_{-1,1}\right)^2}{4\sigma^2_{-1,1}\sigma^2_{1,0}}}}\right) \ldots \end{aligned} \tag{9}$$

[30]Both algorithms are available as a R functions on authors' websites, together with an illustrative example.

$$
\begin{aligned}
&+\frac{\sigma_{-1,0}}{\sqrt{\sigma^2_{-1,0}}} \times 1[m=-1] \frac{\phi\left(\frac{\left[\gamma^h_{-1}-\gamma^h_0\right]' z_{ijt}}{\sqrt{\sigma^2_{-1,0}}}\right)}{BN\left(\frac{\left[\gamma^h_{-1}-\gamma^h_0\right]' z_{ijt}}{\sqrt{\sigma^2_{-1,0}}}, \frac{-\left[\gamma^h_1-\gamma^h_{-1,}\right]' z_{ijt}}{\sqrt{\sigma^2_{-1,1}}}; \frac{-\sigma^2_{1,0}+\sigma^2_{-1,0}+\sigma^2_{-1,1}}{2\sqrt{\sigma^2_{-1,1}\sigma^2_{-1,0}}}\right)} \\
&\qquad \times \Phi\left(\frac{\frac{-\left[\gamma^h_1-\gamma^h_0\right]' z_{ijt}}{\sqrt{\sigma^2_{-1,1}}} - \frac{-\sigma^2_{1,0}+\sigma^2_{-1,0}+\sigma^2_{-1,1}}{2\sqrt{\sigma^2_{-1,1}\sigma^2_{-1,0}}} \frac{\left[\gamma^h_{-1}-\gamma^h_0\right]' z_{ijt}}{\sqrt{\sigma^2_{-1,0}}}}{\sqrt{1-\frac{\left(-\sigma^2_{1,0}+\sigma^2_{-1,0}+\sigma^2_{-1,1}\right)^2}{4\sigma^2_{-1,1}\sigma^2_{-1,0}}}}\right) \\
&+\frac{\sigma_{-1,0}-\sigma_{1,0}}{\sqrt{\sigma^2_{-1,1}}} \times 1[m=-1] \frac{\phi\left(\frac{-\left[\gamma^h_1-\gamma^h_0\right]' Z_{ijt}}{\sqrt{\sigma^2_{-1,1}}}\right)}{BN\left(\frac{\left[\gamma^h_{-1}-\gamma^h_0\right]' Z_{ijt}}{\sqrt{\sigma^2_{-1,0}}}, \frac{-\left[\gamma^h_1-\gamma^h_0\right]' Z_{ijt}}{\sqrt{\sigma^2_{-1,1}}}; \frac{-\sigma^2_{1,0}+\sigma^2_{-1,0}+\sigma^2_{-1,1}}{2\sqrt{\sigma^2_{-1,1}\sigma^2_{-1,0}}}\right)} \\
&\qquad \times \Phi\left(\frac{\frac{\left[\gamma^h_{-1}-\gamma^h_0\right]' z_{ijt}}{\sqrt{\sigma^2_{-1,0}}} - \frac{-\sigma^2_{1,0}+\sigma^2_{-1,0}+\sigma^2_{-1,1}}{2\sqrt{\sigma^2_{-1,1}\sigma^2_{-1,0}}} \frac{-\left[\gamma^h_1-\gamma^h_{-1}\right]' z_{ijt}}{\sqrt{\sigma^2_{-1,1}}}}{\sqrt{1-\frac{\left(-\sigma^2_{1,0}+\sigma^2_{-1,0}+\sigma^2_{-1,1}\right)^2}{4\sigma^2_{-1,1}\sigma^2_{-1,0}}}}\right)
\end{aligned}
$$

$\phi(\cdot)$ is the density and $\Phi(\cdot)$ the cumulative density of the standardized normal; $BN(\cdot)$ the standardized bivariate normal cumulative density function; and γ^h_m are the first step coefficients for option m. Note that the selection correction term for $m=0$ is omitted because we exclude zero price changes from the sample when estimating the second step. Since each model h only identifies $\gamma_1-\gamma_0$ as well as $\gamma_{-1}-\gamma_0$, $\gamma_1-\gamma_{-1}$ is computed indirectly as $(\gamma_1-\gamma_0)-(\gamma_{-1}-\gamma_0)$. The covariance terms σ needed in the selection correction terms are directly retrieved from the above Ω.

$$
\begin{aligned}
\sigma^2_{1,0} &= \sigma^2_{1,0} \\
\sigma^2_{-1,0} &= \sigma^2_{-1,0} \\
\sigma^2_{-1,1} &= \sigma^2_{1,0} + \sigma^2_{-1,0} - 0.5\sigma^2_{-1,0/1,0}
\end{aligned}
$$

This allows us to directly estimate the four selection terms which are ordered first on each row of Equation 9. To estimated parameters are reported in Table 8.